## Appendix A Projection Methodology

The general procedure for *Projections of Education Statistics* to 2015 was to express the variable to be projected as a percent of a "base" variable. These percents were then projected and applied to projections of the "base" variable. For example, the number of 18-year-old college students was expressed as a percent of the 18-year-old population for each year from 1972 through 2004. This enrollment rate was then projected through the year 2015 and applied to projections of the 18-year-old population from the U.S. Census Bureau.

Enrollment projections are based primarily on population projections. Projections of high school graduates and earned degrees conferred are based primarily on enrollment projections.

Exponential smoothing and multiple linear regression are the two major projection techniques used in this publication. Single exponential smoothing is used when the historical data have a basically horizontal pattern. On the other hand, double exponential smoothing is used when the time series is expected to change linearly with time. In general, exponential smoothing places more weight on recent observations than on earlier ones. The weights for observations decrease exponentially as one moves further into the past. As a result, the older data have less influence on these projections. The rate at which the weights of older observations decrease is determined by the smoothing constant selected.

$$P = \alpha X_{1} + \alpha (1 - \alpha) X_{1-1} + \alpha (1 - \alpha)^{2} X_{1-2} + \alpha (1 - \alpha)^{3} X_{1-3} + \dots$$

where:

P = projected value

 $\alpha$  = smoothing constant (0 <  $\alpha$  < 1)

 $X_r$  = observation for time t

This equation illustrates that the projection is a weighted average based on exponentially decreasing weights. For a high smoothing constant, weights for earlier observations decrease rapidly. For a low smoothing constant, decreases

are more moderate. Projections of enrollments and public high school graduates are based on a smoothing constant of  $\alpha = 0.4$ .

The farther apart the observations are spaced in time, the more likely it is that there are changes in the underlying social, political, and economic structure. Since the observations are on an annual basis, major shifts in the underlying process are more likely in the time span of just a few observations than if the observations were available on a monthly or weekly basis. As a result, the underlying process for annual models tends to be less stable from one observation to the next. Another reason for using high smoothing constants for some time series is that most of the observations are fairly accurate, because most observations are population values rather than sample estimates. Therefore, large shifts tend to indicate actual changes in the process rather than noise in the data.

Multiple linear regression is also used in making projections of college enrollment and earned degrees conferred. This technique is used when it is believed that a strong relationship exists between the variable being projected (the dependent variable) and independent variables. However, this technique is used only when accurate data and reliable projections of the independent variables are available.

The functional form primarily used is the multiplicative model. When used with two independent variables, this model takes the form:

$$Y = aX_1^{b_1}X_2^{b_2}$$

This equation can easily be transformed into the linear form by taking the natural log (ln) of both sides of the equation:

$$\ln Y = \ln(a) + b, \ln X, + b, \ln X,$$

The multiplicative model has a number of advantages. Research has found that it is a reasonable way to represent human behavior. Constant elasticities are assumed, which means that a 1 percent change in lnX will lead to a given percent change in lnY. This percent change is equal to b<sub>1</sub>.

And the multiplicative model lends itself easily to "a priori" analysis because the researcher does not have to worry about units of measurement when specifying relationships. In fact, the multiplicative model is considered the standard in economic analyses. For additional information, see *Forecasting: Methods and Applications* by Spiro Makridakis, Steven C. Wheelwright, and Rob J. Hyndman (John Wiley and Sons, 1998, p. 607).

#### Assumptions

All projections are based on underlying assumptions, and these assumptions determine projection results to a large extent. It is important that users of projections understand the assumptions to determine the acceptability of projected time series for their purposes. Descriptions of the primary assumptions upon which the projections of time series are based are presented in table A1.

For some projections, low, middle, and high alternatives are shown. These alternatives reveal the level of uncertainty involved in making projections, and they also point out the sensitivity of projections to the assumptions on which they are based.

The key determinants of higher education enrollment are household income, which represents ability to pay, and an age-specific unemployment rate, which acts as a proxy for opportunity costs faced by students. Agespecific unemployment rates are likely to increase during a weak or pessimistic economy, with the result that the estimated opportunity costs will be lower. This will have a positive impact on higher education enrollment, as students face less attractive alternatives. This will be apparent in the short term, resulting in a potential reversal in the expected pattern across the alternative economic scenarios. As a result, the high alternative projections will be lower than the low alternative projections. However, in the long term, the effect of the per capita income variable dominates the effects of the unemployment rate. This results in a pattern where the high alternative projections are greater than the low alternative projections.

Many of the projections in this publication are demographically based on U.S. Census Bureau middle series projections of the population by age. The population projections developed by the U.S. Census Bureau are based on the 2000 census and the middle series assumptions for the fertility rate, internal migration, net immigration, and mortality rate.

The future fertility rate assumption, which determines projections of the number of births, is one key assumption in making population projections. This assumption plays a major role in determining population projections for the age groups enrolled in nursery school, kindergarten, and elementary grades. The effects of the fertility rate assumption are more pronounced toward the end of the projection period, while the immigration assumptions affect all years.

For enrollments in secondary grades and college, the fertility assumption is of no consequence, since all the population cohorts for these enrollment ranges have already been born. For projections of enrollments in elementary schools, only middle series population projections were considered. Projections of high school graduates are based on projections of the percent of grade 12 enrollment that are high school graduates. Projections of associate's, bachelor's, master's, doctor's, and first-professional degrees are based on projections of college-age populations and college enrollment, by sex, attendance status, level enrolled by student, and type of institution. Projections of college enrollment are also based on disposable income per capita and unemployment rates. The projections of elementary and secondary teachers are based on education revenue receipts from state sources and enrollments. The projections of expenditures of public elementary and secondary schools are based on enrollments and projections of disposable income per capita and various revenue measures of state and local governments. Projections of disposable income per capita and unemployment rates were obtained from the company Global Insight, Inc. Many additional assumptions were made in projecting these variables.

#### **Limitations of Projections**

Projections of time series usually differ from the final reported data due to errors from many sources. This is because of the inherent nature of the statistical universe from which the basic data are obtained and the properties of projection methodologies, which depend on the validity of many assumptions. Therefore, alternative projections are shown for most statistical series to denote the uncertainty involved in making projections. These alternatives are not statistical confidence limits, but instead represent judgments made by the authors as to reasonable upper and lower bounds. The mean absolute percentage error is one way to express the forecast accuracy of past projections. This measure expresses the average value of the absolute value of errors in percentage terms. For example, the mean absolute percentage errors of public school enrollment in grades K-12 for lead times of 1, 2, 5, and 10 years were 0.3, 0.5, 1.2, and 2.5 percent, respectively. For more information on mean absolute percentage errors, see table A-2.

Table A-1. Summary of forecast assumptions to 2015

Variable	Middle alternative	Low alternative	High alternative
Demographic assumptions			-
Population	Projections are consistent	Same as middle alterna-	Same as middle alterna-
	with the Census Bureau	tive	tive
	middle series estimates.		
18- to 24-year-old population	Average annual growth	Same as middle alterna-	Same as middle alterna-
to to It your our population	rate of 0.2%	tive	tive
25- to 29-year-old population	Average annual growth	Same as middle alterna-	Same as middle alterna-
, , , , , , , , , , , , , , , , , , , ,	rate of 1.2%	tive	tive
30- to 34-year-old population	Average annual growth	Same as middle alterna-	Same as middle alterna-
,	rate of 0.6%	tive	tive
35- to 44-year-old population	Average annual decline	Same as middle alterna-	Same as middle alterna-
,	of 0.7%	tive	tive
Economic assumptions			
Disposable income per	Annual percent changes	Annual percent changes	Annual percent changes
capita in constant dollars	range between 2.0% and	range between 1.5% and	range between 2.3% and
	2.4% with an annual	2.5% with an annual	3.2% with an annual
	growth rate of 2.5%	growth rate of 2.2%	growth rate of 3.1%
Education revenue receipts from state	Annual percent changes	Annual percent changes	Annual percent changes
sources per capita in constant dollars	range between 1.7% and	range between 1.5% and	range between 1.8% and
	3.1% with an annual	3.2% with an annual	3.7% with an annual
	growth rate of 2.8%	growth rate of 2.4%	growth rate of 3.6%
Inflation rate	Inflation rate ranges	Inflation rate ranges	Inflation rate ranges
	between 1.5% and 2.7%	between 1.5% and 3.7%	between 1.2% and 2.6%
Unemployment rate (men)			
Ages 18 and 19	Remains between	Remains between	Remains between
	16.3% and 17.2%	15.6% and 16.7%	16.0% and 16.8%
Ages 20 to 24	Remains between	Remains between	Remains between
	9.3% and 9.9%	8.9% and 9.6%	9.1% and 9.6%
Age 25 and over	Remains between	Remains between	Remains between
	3.9% and 4.1%	3.7% and 4.0%	3.8% and 4.0%
Unemployment rate (women)			
Ages 18 and 19	Remains between	Remains between	Remains between
	12.4% and 13.1%	11.9% and 12.9%	12.1% and 12.9%
Ages 20 to 24	Remains between	Remains between	Remains between
	7.9% and 8.4%	7.6% and 8.3%	7.7% and 8.3%
Age 25 and over	Remains between	Remains between	Remains between
	3.9% and 4.1%	3.7% and 4.1%	3.8% and 4.1%

SOURCE: U.S. Department of Commerce, Census Bureau, previously unpublished tabulation (June 2004); and Global Insight, Inc., "U.S. Quarterly Model." (This table was prepared December 2005.)

Table A-2. Mean absolute percentage errors (MAPEs) by lead time for selected statistics in all public elementary and secondary schools and degree-granting institutions: 2005

				I	ead time	(years)				
Statistic	1	2	3	4	5	6	7	8	9	10
Public elementary and secondary schools										
PK-12 enrollment	0.3	0.5	0.8	1.0	1.2	1.3	1.5	1.8	2.1	2.5
PK-8 enrollment	0.4	0.6	0.9	1.1	1.2	1.4	1.8	2.3	2.9	3.5
9–12 enrollment	0.4	0.7	1.0	1.2	1.3	1.5	1.8	2.2	2.3	2.3
High school graduates	0.8	0.9	1.6	1.7	1.4	1.6	2.4	3.5	4.0	3.9
Elementary and secondary teachers	1.0	1.6	1.8	2.2	2.7	3.4	4.0	4.5	4.7	5.6
Total current expenditures <sup>1</sup>	1.3	2.5	2.3	2.4	2.9	4.0	4.8	4.1	3.7	3.0
Current expenditures per pupil in fall enrollment <sup>1</sup>	1.3	2.3	2.1	2.3	3.5	4.5	5.1	4.8	5.5	5.2
Estimated average annual teacher salaries <sup>1</sup>	1.2	1.6	2.0	3.4	5.0	6.3	7.9	9.0	9.5	9.7
Degree-granting institutions										
Total enrollment	1.5	2.4	3.1	3.6	4.6	6.3	8.2	9.8	10.1	_
Men	1.6	2.8	3.4	4.2	5.7	7.1	8.5	9.6	9.2	_
Women	1.7	2.7	3.7	3.6	3.8	5.6	7.9	10.0	10.8	_
4-year institutions	1.1	2.0	2.8	3.9	4.5	5.8	8.0	9.8	10.5	_
2-year institutions	2.4	3.9	4.4	4.3	5.2	7.0	8.4	9.8	9.4	_
Associate's degrees	2.3	2.9	2.9	4.9	5.6	6.7	7.5	8.7	11.3	12.4
Bachelor's degrees	0.9	2.0	2.8	3.7	5.7	7.5	8.3	8.8	9.4	9.7
Master's degrees	1.6	4.1	7.7	9.9	11.3	14.2	16.4	16.7	15.7	17.4
Doctor's degrees	2.6	3.5	3.0	3.7	2.5	2.3	4.8	5.2	1.7	2.6
First-professional degrees	1.3	1.3	1.8	3.4	5.5	7.0	8.8	10.6	10.5	10.0

<sup>—</sup>Not available. Not all actual values were available to calculate a MAPE for this lead time.

NOTE: Mean absolute percentage error is the average value of the absolute values of errors expressed in percentage terms. MAPEs for K-12 enrollments were calculated using the last 22 editions of *Projections of Education Statistics*. MAPEs for high school graduates were calculated from the past 15 editions of *Projections of Education Statistics*. MAPEs for teachers were calculated from the past 15 editions containing teachers projections and MAPEs for current expenditures and teacher salaries were calculated using projections from the last 15 editions containing current expenditure and teacher salary projections. MAPEs for degree-granting institution enrollments and earned degrees were calculated using the last 8 and 9 editions, respectively. MAPEs for current-fund expenditures were calculated using the last 10 editions of *Projections of Education Statistes* that included projections of current-fund expenditures. Calculations were made using unrounded numbers. Some data have been revised from previously published numbers.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Projections of Education Statistics, various issues. (This table was prepared November 2005.)

<sup>&</sup>lt;sup>1</sup>In constant dollars based on the Consumer Price Index for all urban consumers, Bureau of Labor Statistics, U.S. Department of Labor.

### **Enrollment**

#### **National**

Enrollment projections are based on projected enrollment rates, by age and sex, where the enrollment rate for a given population for a certain level of education is the number of people in that population enrolled at that level of education divided by the total number of people in that population. These enrollment rates were projected by taking into account the most recent trends, as well as the effects of economic conditions and demographic changes. The projected enrollment rates were then used in the Education Forecasting Model (EDMOD), which consists of age-specific rates by sex and by enrollment levels.

Enrollments by age and age groups from the U.S. Census Bureau were adjusted to NCES totals to compute rates for 1972 through 2004. The first stage of EDMOD is an age-specific enrollment model in which these enrollment rates are projected and applied to age-specific population projections from the U.S. Census Bureau. This stage includes all ages for students enrolled in grades K–12 and for students enrolled in colleges and universities. This stage, which is used separately for each sex, consists of the following categories: (1) nursery and kindergarten; (2) elementary grades 1–8; (3) secondary grades 9–12; (4) full-time college enrollment; and (5) part-time college enrollment.

At the postsecondary level, projections of full-time and part-time college enrollments were considered only for ages 16 and over. College enrollment is negligible for earlier ages. Full-time and part-time enrollments are modeled separately, with each model run by sex. Within an enrollment category, where applicable, college enrollment rates were projected by individual ages 16 through 24 and for the age groups 25 to 29, 30 to 34, and 35 years and over. Three alternative projections were made using various economic assumptions. Table A-3 shows enrollment rates for 2004 and middle alternative projected enrollment rates for 2010 and 2015. Table A-4 shows the equations used to project the enrollments for men by attendance status. Table A-5 shows the equations used to project enrollment rates for women by attendance.

#### Enrollment in Public Elementary and Secondary Schools, by Grade Group and Organizational Level

The second stage of EDMOD projects public enrollment in elementary and secondary schools by grade group and by organizational level. Public enrollments by age were based on enrollment rate projections for nursery and kindergarten, grade 1, elementary ungraded and special, and secondary ungraded and special. Grade progression rate projections were used for grades 2 through 12. Table A-6 shows the public school enrollment rates, and table A-7 shows the public school grade progression rates for 2003 and projections for 2004 through 2015. The projected rates in tables A-6 and A-7 were used to compute the projections of enrollments in elementary and secondary schools, by grade, shown in table 3.

## College Enrollment, by Sex, Attendance Status, and Level Enrolled, and by Type and Control of Institution

The third stage of EDMOD projects enrollments in degree-granting institutions, by age group, sex, attendance status, and level enrolled by student, and by type and control of institution. These projections for 2005 through 2015 are shown in tables A-8 and A-9, along with actual values for 2004. For all projections, it was assumed that there was no enrollment in 2-year institutions at the postbaccalaureate level (graduate and first-professional).

The projected rates in tables A-8 and A-9 were then adjusted to agree with the projected age-specific enrollment rates in the first stage of EDMOD. The adjusted rates were then applied to the projected enrollments by age group, sex, and attendance status from the first stage of EDMOD to obtain projections by age group, sex, attendance status, level enrolled, and type of institution.

For each enrollment category—sex, attendance status, level enrolled, and type of institution—public enrollment was projected as a percent of total enrollment. Projections for 2005 through 2015 are shown in table A-10, along with actual percents for 2004. The projected rates were then applied to the projected enrollments in each enrollment category to obtain projections by control of institution.

For each category by sex, enrollment level, and type and control of institution, graduate enrollment was projected as a percent of postbaccalaureate enrollment. Actual rates for 2004 and projections for 2005 through 2015 are shown in table A-11. The projected rates in table A-11 were then applied to projections of postbaccalaureate enrollment to obtain graduate and first-professional enrollment projections by sex, attendance status, and type and control of institution.

#### Full-Time-Equivalent Enrollment, by Type and Control of Institution and by Level Enrolled

The fourth stage of EDMOD projects full-time-equivalent enrollment, by type and control of institution and by level enrolled. The full-time-equivalent enrollment measures enrollment as if students were enrolled full time for one academic year, and equals the sum of full-time enrollment and full-time-equivalent of part-time enrollment. The full-time-equivalent of part-time enrollment was estimated as a percentage of part-time enrollment. In EDMOD, the full-time-equivalent of part-time enrollment was calculated using different percentages for enrollment category by level enrolled and by type and control of institution. Actual percents for 2004 and projections for 2005 and 2015 are shown in table A-12.

These projected percents were applied to part-time projections of enrollment by level enrolled and by type and control of institution from the third stage of EDMOD. These equivalent of part-time projections were added to projections of full-time enrollment (from the previous stage) to obtain projections of full-time-equivalent enrollment.

## College Enrollment, by Sex, Attendance Status, Age Group, and Race/Ethnicity

The fifth stage of EDMOD projects enrollments in degreegranting institutions by age, sex, attendance status, and race/ethnicity. The race/ethnicity groups projected include the following: White, Non-Hispanic; Black, Non-Hispanic; Hispanic; Asian or Hawaiian-Pacific Islander, Non-Hispanic; American Indian/Alaska Native, Non-Hispanic; and Non-Resident Alien. Enrollment projections are based on projected enrollment rates by age, sex, attendance status, and race/ethnicity where the enrollment rate for a given population for a certain level of education is the number of people in that population enrolled at that level of education divided by the total number of people in that population. With the exception of American Indian/Alaska Native, Non-Hispanic and Non-Resident Alien, all race/ ethnicity groups were projected by taking into account the most recent trends, as well as the effects of economic conditions and demographic changes. Due to the nature of the historical data, American Indian/Alaska Native, Non-Hispanic enrollments were projected using single exponential smoothing and Non-Resident Alien enrollments were projected using patterns in recent historical growth.

Enrollments by sex, race/ethnicity and age from the U.S. Census Bureau were adjusted to NCES totals by sex and race/ethnicity to compute rates for 1980 through 2004. As

with the first stage of EDMOD, the fifth stage consists of age-specific enrollment models for each sex-race/ethnicity group in which enrollment rates are projected and applied to age-specific population projections by sex and race/ethnicity from the U.S. Census Bureau. The final set of projected rates by age, sex, attendance status, and race/ethnicity were controlled to the stage one enrollment rates by age, sex, and attendance status to ensure consistency across stages.

Stage five consists of sixteen individual pooled time series models—one for each attendance status - sex - race/ethnicity combination—that are each pooled across age. As with the stage one postsecondary level projections, projections of full-time and part-time college enrollments by race/ ethnicity were considered only for ages 16 and over. College enrollment is negligible for earlier ages. Within each model, college enrollment rates were projected by individual ages 16 through 24 and for the age groups 25 to 29, 30 to 34, and 35 years and over. Table A-14 shows the equations used to project the enrollments for White, Non-Hispanic men by attendance status. Table A-15 shows the equations used to project enrollment rates for White, Non-Hispanic women by attendance. Table A-16 shows the equations used to project the enrollments for Black, Non-Hispanic men by attendance status. Table A-17 shows the equations used to project enrollment rates for Black, Non-Hispanic women by attendance. Table A-18 shows the equations used to project the enrollments for Hispanic men by attendance status. Table A-19 shows the equations used to project enrollment rates for Hispanic women by attendance. Table A-20 shows the equations used to project the enrollments for Asian or Hawaiian-Pacific Islander, Non-Hispanic men by attendance status. Table A-21 shows the equations used to project enrollment rates for Asian or Hawaiian-Pacific Islander, Non-Hispanic women by attendance status.

#### **Projection Accuracy**

An analysis of projection errors from the past 22 editions of *Projections of Education Statistics* indicates that the mean absolute percentage errors (MAPEs) for lead times of 1, 2, 5, and 10 years out for projections of public school enrollment in grades K–12 were 0.3, 0.5, 1.2, and 2.5 percent, respectively. For the 1-year-out prediction, this means that one would expect the projection to be within 0.3 percent of the actual value, on the average. For projections of public school enrollment in grades K–8, the MAPEs for lead times of 1, 2, 5, and 10 years out were 0.4, 0.6, 1.2, and 3.5 percent, respectively, while those for projections of public school enrollment in grades 9–12 were 0.4, 0.7, 1.3, and 2.3 percent for the same lead times.

For projections of total enrollment in degree-granting institutions, an analysis of projection errors based on the past 8 editions of *Projections of Education Statistics* indicates that the MAPEs for lead times of 1, 2, and 5 years were 1.5, 2.4, and 4.6 percent, respectively. For the 1-year-out prediction, this means that one would expect the projection to be within 1.5 percent of the actual value, on the average. For more information on MAPEs, see table A-2, page 89.

#### **Basic Methodology**

The notation and equations that follow describe the basic models used to project public elementary and secondary enrollment.

#### **Public Elementary and Secondary Enrollment**

#### Let:

- i = Subscript denoting age
- j = Subscript denoting grade
- t = Subscript denoting time
- K<sub>t</sub> = Enrollment at the nursery and kindergarten level
- G<sub>ir</sub> = Enrollment in grade j
- $G_{1r}$  = Enrollment in grade 1
- E<sub>t</sub> = Enrollment in elementary special and ungraded programs
- $S_{t}$  = Enrollment in secondary special and ungraded programs
- P<sub>ir</sub> = Population age i
- $RK_{t}$  = Enrollment rate for nursery and kindergarten
- $RG_{1r}$  = Enrollment rate for grade 1

- RE<sub>t</sub> = Enrollment rate for elementary special and ungraded programs
- RS<sub>t</sub> = Enrollment rate for secondary special and ungraded programs
- EG<sub>.</sub> = Total enrollment in elementary grades (K–8)
- SG<sub>2</sub> = Total enrollment in secondary grades (9–12)
- R<sub>jt</sub> = Progression rate for grade j: the proportion that enrollment in grade j in year t is of enrollment in grade j 1 in year t-1.

#### Then:

$$EG_{i} = K_{i} + E_{i} + \sum_{j=1}^{8} G_{ji}$$

$$SG_t = S_t + \sum_{j=0}^{12} G_{jt}$$

#### where:

$$K_r = RK_r(P_{cr})$$

$$G_{jt} = R_{jt} \left( G_{j-l,t-l} \right)$$

$$E_{t} = RE_{t} \left( \sum_{i=5}^{13} P_{it} \right)$$

$$G_{1t} = RG_{it}(P_{6t})$$

$$S_{t} = RS_{t} \left( \sum_{i=14}^{17} P_{it} \right)$$

#### **Enrollment in Degree-Granting Institutions**

For degree-granting institutions, projections were computed separately by sex and attendance status of student. The notation and equations are:

#### Let:

i = Subscript denoting age except:

i = 25: ages 25–29

i = 26: ages 30–34

i = 27: ages 35 and over for enrollment (35–44 for population)

t = Subscript denoting year

j = Subscript denoting sex

k = Subscript denoting attendance status

 $E_{ijkt}$  = Enrollment of students age i by sex and attendance status

P<sub>iit</sub> = Population age i by sex

R<sub>ijkt</sub> = Enrollment rate for students age i by sex and attendance status

T<sub>ijkt</sub> = Total enrollment for particular subset of students: full-time men, full-time women, part-time men, part-time women

#### Then:

$$T_{ijkr} = \sum_{i=16}^{27} E_{ijkr}$$

where:

$$E_{nor} = R_{nor}(P_{nr})$$

## Enrollment in Degree-Granting Institutions by Race/Ethnicity

With this edition of the *Projections of Education Statistics*, projections for degree-granting institutions by sex and attendance status of student were further disaggregated by race/ethnicity for the first time. The notation and equations are:

#### Let:

= Subscript denoting age except:

i = 25: ages 25–29

i = 26: ages 30–34

i = 27: ages 35 and over for enrollment (35–44 for population)

t = Subscript denoting year

j = Subscript denoting sex

k = Subscript denoting attendance status

1 = Subscript denoting race/ethnicity

E<sub>ijklt</sub> = Enrollment of students age i by sex, attendance status, and race/ethnicity

P<sub>iilt</sub> = Population age i by sex and race/ethnicity

R<sub>ijklt</sub> = Enrollment rate for students age i by sex, attendance status, and race/ethnicity

 $\Gamma_{ijklt}$  = Total enrollment for a particular subset of students by race/ethnicity: full-time men, full-time women, part-time men, part-time women

#### Then:

$$T_{ijkh} = \sum_{i=16}^{27} E_{ijkh}$$

where:

$$E_{min} = R_{min} (P_{mb})$$

#### **Methodological Tables**

Table A-22 gives the basic assumptions underlying enrollment projections.

#### **Private School Enrollment**

This edition is the fifth report that projected trends in elementary and secondary enrollment by grade level in private schools using the grade progression rate method.

Private school enrollment data from the NCES Private School Universe Survey for 1989–90, 1991–92, 1993–94, 1995–96, 1997–98, 1999–2000, and 2001–02 were used to develop these projections. In addition, population estimates for 1989 to 2004 and population projections for 2005 to 2015 from the U.S. Census Bureau were used to develop the projections.

Prekindergarten, kindergarten, and first-grade enrollments are based on projected enrollment rates of 5- and 6-year-olds. These projected enrollment rates are applied to population projections of 5- and 6-year-olds developed by the U.S. Census Bureau.

Enrollments in grades 2 through 12 are based on projected grade progression rates. The grade progression rate method starts with 6-year-olds entering first grade and then follows their progress through private elementary and secondary schools. The method requires calculating the ratio of the number of children in one year who "survive" the year and enroll in the next grade the following year. These projected rates are then applied to the current enrollment by grade to yield grade-by-grade projections for future years.

Enrollment rates of 5- and 6-year-olds and grade progression rates are projected using single exponential smoothing. Elementary ungraded and secondary ungraded are projected to remain constant at their 2001 levels. To obtain projections of total enrollment, projections of enrollments for the individual grades (prekindergarten through 12) and ungraded were summed.

The grade progression rate method assumes that past trends in factors affecting private school enrollments will continue over the projection period. This assumption implies that all factors influencing enrollments will display future patterns consistent with past patterns. This method implicitly includes the net effect of such factors as migration, dropouts, deaths, nonpromotion, and transfers to and from public schools.

Mean absolute percentage errors (MAPEs) of the projection accuracy of private school enrollment were not developed because this projection method has been developed only recently and there is not yet enough historical information to evaluate model performance. As additional data become available, MAPEs can then be calculated.

#### State Level

For the 50 states and the District of Columbia, this edition contains projected trends in elementary and secondary enrollment by grade level in public schools from 2004 to the year 2015. This is the 11th report on state-level projections for public school elementary and secondary education statistics.

Public school enrollment data from the NCES Common Core of Data survey for 1980 to 2003 were used to develop these projections. This survey does not collect enrollment data for private schools.

Population estimates for 1980 to 2004 and population projections for 2005 to 2015 from the U.S. Census Bureau were used to develop the enrollment projections. The state population projections used in this year's update have been revised relative to last year's update. The set of population projections used in this year's Projections of Education Statistics to 2015 are the Census Bureau's newly released (April 2005) set of interim state-level population projections. This set of state-level projections line up with the Census Bureau's interim national population projections, which were released earlier in May 2004. The population projections used in last year's Projections of Education Statistics were based on the Census Bureau's old state-level population projections, but were adjusted to line up with the 2002 state-level population estimates and interim national population projections. During the next year, the Census Bureau plans to develop a revised set of population projections that will be consistent with a revised set of national population projections and that will include modifications to produce projections by race and Hispanic origin as well as by age and sex.

The changes in the underlying population projections impact the final state-level enrollment projections in this year's edition of the *Projections of Education Statistics*. While the impact varies by state, this year's state-level projections are substantially different than the state-level projections released in last year's publication, *Projections of Education Statistics to 2014*.

Table A-13 describes the number of years, projection methods, and smoothing constants used to project enrollments in public schools. Also included in table A-13 is the procedure for choosing the different smoothing constants for the time-series models.

All states, with the exception of the District of Columbia, were projected using the same single exponential smoothing parameter. Due to questions about the quality of the District of Columbia data, the smoothing parameters for the District of Columbia were estimated using the available historical data. This approach yielded more consistent projections of the District of Columbia enrollments.

Projections of enrollment in public elementary and secondary schools by state were developed using primarily the grade progression rate method. Prekindergarten, kindergarten, and first-grade enrollments are based on projected enrollment rates of 5- and 6-year-olds. These projected enrollment rates are applied to population projections of 5- and 6-year-olds developed by the U.S. Census Bureau.

Enrollments in grades 2 through 12 are based on projected grade progression rates in each state. These projected rates are then applied to the current enrollment by grade to yield grade-by-grade projections for future years. Enrollment rates of 5- and 6-year-olds and grade progression rates are projected using single exponential smoothing. Elementary ungraded and secondary ungraded

are projected to remain constant at their 2003 levels. To obtain projections of total enrollment, projections of enrollments for the individual grades (kindergarten through 12) and ungraded were summed.

The grade progression rate method assumes that past trends in factors affecting public school enrollments will continue over the projection period. This assumption implies that all factors influencing enrollments will display future patterns consistent with past patterns. Therefore, this method has limitations when applied to states with unusual changes in migration rates. This method implicitly includes the net effect of such factors as migration, dropouts, deaths, nonpromotion, and transfers to and from private schools.

#### Adjustment to National Projections

The projections of state enrollments were adjusted to sum to the national projections of public school K–12, K–8, and 9–12 enrollments shown in table 1. For details on the methods used to develop the national projections for this statistic, see the section on national enrollment projections in this appendix.

Table A-3. Actual and middle alternative projected numbers for college enrollment rates, by sex, attendance status, and age: Fall 2004, 2010, and 2015

		Projected	
Sex, attendance status, and age	Actual 2004	2010	2015
Men			
Full-time			
16 years old	0.2	0.3	0.3
17 years old	2.3	2.4	2.6
18 years old	26.0	26.9	28.3
19 years old	35.5	36.5	38.2
20 years old	34.1	35.1	36.6
21 years old	30.4	31.4	32.9
22 years old	21.1	21.9	23.1
23 years old	12.0	12.5	13.3
24 years old	10.7	11.2	11.9
25 to 29 years old	5.1	5.3	5.7
30 to 34 years old	1.9	2.0	2.2
35 to 44 years old	1.4	1.5	1.6
Part-time			
16 years old	#	#	#
17 years old	0.7	0.7	0.8
18 years old	4.7	4.8	4.9
19 years old	6.5	6.6	6.7
20 years old	9.1	9.2	9.3
21 years old	7.6	7.7	7.9
22 years old	5.3	5.4	5.6
23 years old	7.0	7.2	7.5
24 years old	6.8	6.9	7.2
25 to 29 years old	5.3	5.5	5.7
30 to 34 years old	3.1	3.1	3.3
35 to 44 years old	1.4	1.5	1.6
Women		1.7	1.0
Full-time			
16 years old	0.3	0.5	0.5
17 years old	4.3	3.4	4.2
18 years old	39.4	42.6	44.1
19 years old	45.9	49.1	50.5
20 years old	40.0	43.2	44.5
21 years old	36.6	39.7	41.0
22 years old	24.3	26.8	28.0
23 years old	16.1	18.0	21.3
24 years old	13.6	15.3	18.2
25 to 29 years old	6.5	7.4	8.9
30 to 34 years old	2.9	3.3	4.0
35 to 44 years old	2.3	2.6	3.2
Part-time	2.5	2.0	5.2
16 years old	0.1	0.1	0.1
17 years old	1.0	1.0	1.1
18 years old	6.1	6.0	6.3
19 years old	9.4	9.2	9.5
20 years old	9.2	9.0	9.4
21 years old	10.0	10.0	10.4
22 years old	8.8	8.9	9.4
	8.0	8.2	8.4
23 years old	9.5	9.8	10.1
25 to 29 years old	7.5	7.8	8.2
	7.3 5.2	7.8 5.4	5.7
30 to 34 years old			
35 to 44 years old	6.9	7.2	7.7

<sup>#</sup> Rounds to zero

Table A-4. Equations for full-time and part-time college enrollment rates of men

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-5.61	0.26	-21.5	0.99	2.19
Age 18	-3.00	0.22	-13.6		
Age 19	-2.76	0.18	-14.9		
Age 20	-2.91	0.19	-15.7		
Age 21	-3.02	0.19	-16.2		
Age 22	-3.53	0.19	-18.7		
Age 23	-3.94	0.19	-21.2		
Age 24	-4.24	0.20	-21.7		
Age 25	-5.04	0.21	-23.8		
Age 25-29	-6.00	0.20	-29.4		
Age3 5-44	-6.62	0.20	-33.5		
LNRYPDRNMA	0.41	0.04	11.6		
LNRUM	0.10	0.04	2.8		
Rho17	0.70	0.10	7.3		
Rho18	0.81	0.08	10.4		
Rho19	0.27	0.14	2.0		
Rho20	0.36	0.14	2.5		
Rho21	0.36	0.14	2.5		
Rho22	0.42	0.13	3.3		
Rho23	0.07	0.14	0.5		
Rho24	0.66	0.10	6.8		
Rho25-29.	0.81	0.07	12.2		
Rho30-34	0.63	0.07	5.9		
Rho35-44.	0.38	0.11	3.6		
Part-time	0.36	0.11	5.0		
	-7.62	0.92	-8.3	0.37	1.78
Age 17	-4.07	0.66	-6.2	0.5/	1./0
Age 19	-3.69	0.72	-5.1		
e	-3.69	0.66	-5.6		
Age 20	-3.78	0.66	-5.7		
Age 21					
Age 22	-3.67	0.66	-5.5		
Age 23	-3.94	0.66	-6.0		
Age 24	-4.14	0.68	-6.1		
Age 25	-4.19	0.68	-6.2		
Age 25-29	-4.62	0.69	-6.7		
Age 35-44	-4.67	0.67	-7.0		
LNRYPDRNMA	0.25	0.12	2.1		
LNRUM	0.02	0.08	0.3		
Rho17	-0.20	0.16	-1.3		
Rho18	0.25	0.18	1.4		
Rho19	0.86	0.08	11.4		
Rho20	0.46	0.15	3.0		
Rho21	0.43	0.17	2.6		
Rho22	0.44	0.16	2.7		
Rho23	0.33	0.18	1.8		
Rho24	0.70	0.12	6.1		
Rho25-29	0.76	0.08	9.1		
Rho30-34	0.83	0.07	11.2		
Rho35-44	0.58	0.15	3.8		

 $R^2$  = Coefficient of determination.

AGE(age) = Enrollment rate by age.

Rho(age) = Autocorrelation coefficient for each age.

LNRUM = Log unemployment rate for men.

LNRYPDRNMA = Log of three-period weighted average of per capita real disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method with a first-order autocorrelation correction. The time period used to estimate the equations is from 1975 to 2004. The number of observations is 352. For additional information, see M. D. Intriligator, *Econometric* Models, Techniques, & Applications, New Jersey: Prentice-Hall, Inc., 1978, pp. 165-173.

D.W. statistic = Durbin-Watson statistic.

Table A-5. Equations for full-time and part-time college enrollment rates of women

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-9.90	1.48	-6.7	0.99	2.37
Age 18	-6.57	0.25	-26.6		
Age 19	-6.40	0.19	-33.4		
Age 20	-6.58	0.19	-35.2		
Age 21	-6.79	0.19	-36.3		
Age 22	-7.52	0.21	-35.2		
Age 23	-7.95	0.20	-39.2		
Age 24	-8.23	0.19	-43.3		
Age 25	-8.92	0.19	-45.9		
Age 25-29	-9.64	0.19	-51.0		
Age 35-44	-9.88	0.19	-52.7		
LNRYPDRNMA	1.16	0.04	26.9		
LNRUM	0.27	0.06	4.6		
Rho17	0.96	0.06	16.9		
Rho18	0.85	0.07	11.6		
Rho19	0.29	0.14	2.1		
Rho20	0.23	0.14	1.6		
Rho21	0.27	0.14	1.9		
Rho22	0.75	0.07	10.0		
Rho23	0.70	0.09	8.2		
Rho24	0.43	0.07	4.1		
Rho25-29	0.66	0.11	6.9		
Rho30-34	0.37	0.10	2.8		
Rho35-44.					
Part-time	0.03	0.12	0.3		
	-7.29	0.56	-12.9	0.78	2.2
Age 17				0./8	2.2
Age 18	-4.51	0.37	-12.3		
Age 19	-4.21	0.42	-10.0		
Age 20	-4.28	0.37	-11.6		
Age 21	-4.35	0.40	-10.8		
Age 22	-4.31	0.35	-12.2		
Age 23	-4.60	0.36	-12.8		
Age 24	-4.70	0.39	-12.2		
Age 25	-4.88	0.34	-14.2		
Age 25-29	-5.10	0.35	-14.4		
Age 35-44	-4.88	0.35	-14.1		
LNRYPDRNMA	0.40	0.08	5.2		
LNRUM	0.01	0.07	0.1		
Rho17	0.45	0.12	3.7		
Rho18	0.45	0.17	2.6		
Rho19	0.81	0.08	9.7		
Rho20	0.60	0.14	4.4		
Rho21	0.78	0.08	9.2		
Rho22	0.28	0.14	2.1		
Rho23	0.52	0.12	4.2		
Rho24	0.77	0.10	7.9		
Rho25-29	0.43	0.13	3.4		
Rho30-34	0.76	0.08	9.9		
Rho35-44	0.58	0.11	5.4		

 $R^2$  = Coefficient of determination.

AGE(age) = Enrollment rate by age.

Rho(age) = Autocorrelation coefficient for each age.

LNRUM = Log unemployment rate for men.

LNRYPDRNMA = Log of three-period weighted average of per capita real disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method with a first-order autocorrelation correction. The time period used to estimate the equations is from 1975 to 2004. The number of observations is 352. For additional information, see M. D. Intriligator, Econometric Models, Techniques, & Applications, New Jersey: Prentice-Hall, Inc., 1978, pp. 165-173.

D.W. statistic = Durbin-Watson statistic.

Table A-6. Actual and projected numbers for enrollment rates in public schools, by grade level: Fall 2003, and 2004 through 2015

Grade level	Actual 2003	Projected 2004 through 2015
Prekindergarten	21.4	22.7
Kindergarten	87.1	88.7
Grade 1	91.2	92.0
Elementary ungraded	0.9	0.8
Secondary ungraded	0.9	0.8

NOTE: The the base age for each grade level is as follows: kindergarten, 5 years old; grade 1, 6 years old; elementary ungraded, 5- to 13-years-olds; and secondary ungraded 14- to 17-years-olds.

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Elementary and Secondary Enrollment Model, 1972–2003. (This table was prepared November 2005.)

Table A-7. Actual and projected numbers for public school grade progression rates: Fall 2003, and 2004 through 2015

Grade	Actual 2003	Projected 2004 through 2015
1 to 2	98.5	98.6
2 to 3	100.6	100.9
3 to 4	100.3	100.2
4 to 5	100.4	100.4
5 to 6	101.6	101.6
6 to 7	101.5	101.4
7 to 8	99.5	99.6
8 to 9	113.3	113.2
9 to 10	89.1	89.3
10 to 11	90.8	91.1
11 to 12	93.1	93.6

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Elementary and Secondary Enrollment Model, 1972–2003. (This table was prepared November 2005.)

Actual and projected numbers for the percentage distribution of full-time students at degree-granting postsecondary institutions, for each age and sex classification: Fall 2004, and 2005 through 2015

	Men		Wome	n
Age	Actual 2004	Projected 2005 through 2015	Actual 2004	Projected 2005 through 2015
Undergraduate, 4-year institutions				
16 and 17 years old	66.7	60.6	64.1	63.8
18 and 19 years old	64.4	65.2	68.1	67.7
20 and 21 years old	76.7	76.8	78.3	78.7
22 to 24 years old	63.8	63.2	61.2	61.0
25 to 29 years old	41.7	41.5	39.3	38.8
30 to 34 years old	37.7	39.5	35.7	32.7
35 years and over	37.8	37.3	35.3	37.7
Undergraduate, 2-year institutions				
16 and 17 years old	32.2	36.9	34.2	32.6
18 and 19 years old	34.9	34.2	31.1	31.8
20 and 21 years old	21.4	20.9	19.3	19.2
22 to 24 years old	18.0	17.9	18.2	16.9
25 to 29 years old	17.5	19.9	27.2	27.2
30 to 34 years old	22.1	20.2	34.3	36.7
35 years and over	27.8	29.7	35.8	35.6
Postbaccalaureate, 2-year institutions				
16 and 17 years old	1.1	2.5	1.6	3.5
18 and 19 years old	0.7	0.6	0.8	0.5
20 and 21 years old	1.9	2.2	2.3	2.0
22 to 24 years old	18.1	18.9	20.6	22.1
25 to 29 years old	40.8	38.7	33.5	34.1
30 to 34 years old	40.2	40.3	29.9	30.6
35 years and over	34.4	33.1	28.9	26.7

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions Model, 1980–2004. (This table was prepared November 2005.)

Actual and projected numbers for the percentage distribution of part-time students at degree-granting postsecondary institutions, for each age and sex classification: Fall 2004, and 2005 through 2015

	Men		Wome	n
Age	Actual 2004	Projected 2005 through 2015	Actual 2004	Projected 2005 through 2015
Undergraduate, 4-year institutions				_
16 and 17 years old	14.0	8.4	24.8	16.7
18 and 19 years old	17.2	17.2	21.7	20.6
20 and 21 years old	28.1	25.4	30.3	28.6
22 to 24 years old	26.6	28.8	24.7	27.9
25 to 29 years old	28.6	30.1	25.8	25.4
30 to 34 years old	28.6	26.3	24.4	24.8
35 years and over	20.6	21.2	21.0	21.0
Undergraduate, 2-year institutions				
16 and 17 years old	85.8	91.5	75.1	83.2
18 and 19 years old	82.5	82.6	77.8	79.1
20 and 21 years old	71.0	73.9	68.9	71.0
22 to 24 years old	65.6	62.8	65.6	61.1
25 to 29 years old	50.6	49.5	50.2	50.1
30 to 34 years old	44.5	45.2	51.8	52.9
35 years and over	52.6	51.9	54.0	54.5
Postbaccalaureate, 2-year institutions				
16 and 17 years old	0.1	0.1	0.1	0.1
18 and 19 years old	0.4	0.2	0.5	0.3
20 and 21 years old	1.0	0.7	0.7	0.4
22 to 24 years old.	7.7	8.4	9.8	11.0
25 to 29 years old	20.8	20.3	24.0	24.5
30 to 34 years old	27.0	28.5	23.8	22.3
35 years and over	26.8	26.9	25.0	24.5

NOTE: Detail may not sum to totals because of rounding.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions Model, 1980–2004. (This table was prepared November 2005.)

Table A-10. Actual and projected numbers for enrollment in public degree-granting postsecondary institutions as a percent of total enrollment, by sex, attendance status, level enrolled, and type of institution: Fall 2004, and 2005 through 2015

	Men		Women		
Enrollment category	Actual 2004	Projected 2005 through 2015	Actual 2004	Projected 2005 through 2015	
Full-time, undergraduate, 4-year institutions	66.9	66.5	65.7	65.0	
Part-time, undergraduate, 4-year institutions	70.8	70.4	68.6	68.3	
Full-time, undergraduate, 2-year institutions	90.7	91.0	90.9	90.4	
Part-time, undergraduate, 2-year institutions	99.1	99.1	98.9	98.8	
Full-time, postbaccalaureate, 4-year institutions	52.3	51.4	52.6	51.3	
Part-time, postbaccalaureate, 4-year institutions	57.1	56.6	61.1	60.1	

SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions Model, 1980–2004. (This table was prepared November 2005.)

Table A-11. Actual and projected numbers for graduate enrollment in degree-granting postsecondary institutions as a percent of total post baccalaureate enrollment, by sex, attendance status, and type and control of institution: Fall 2004, and 2005 through 2015

	Men		Women		
Enrollment category	Actual 2004	Projected 2005 through 2015	Actual 2004	Projected 2005 through 2015	
Full-time, 4-year, public	79.0	79.2	81.0	81.0	
Part-time, 4-year, public	98.8	98.8	99.3	99.3	
Full-time, 4-year, private	67.7	68.7	74.6	75.8	
Part-time, 4-year, private	92.2	92.4	95.5	95.6	

SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions Model, 1980–2004. (This table was prepared November 2005.)

Table A-12. Actual and projected numbers for full-time-equivalent enrollment in degree-granting postsecondary institutions as a percent of part-time enrollment, by type and control of institution level, and level enrolled: Fall 2004, and 2005 through 2015

Enrollment category	Actual 2004	Projected 2005 through 2015
Public, 4-year, undergraduate	40.4	40.4
Public, 2-year, undergraduate	33.6	33.6
Private, 4-year, undergraduate	39.3	39.3
Private, 2-year, undergraduate	39.7	39.7
Public, 4-year, graduate	36.2	36.2
Private, 4-year, graduate	38.2	38.2
Public, 4-year, first-professional	60.1	60.1
Private, 4-year, first-professional	54.6	54.6

SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions Model, 1980–2004. (This table was prepared November 2005.)

Table A-13. Number of years, projection methods, and smoothing constants used to project state-level public school enrollments and high school graduates

Projected state variable	Number of years (1972–2003)	,	Smoothing constant <sup>1</sup>	Basis for smoothing constant
Grade progression rates	31	Single exponential smoothing	0.4	Empirical research
Graduates/grade 12 enrollment	31	Single exponential smoothing	0.4	Empirical research

<sup>1</sup>Alternative smoothing constants were used for the District of Columbia.

SOURCE: U.S. Department of Education, National Center for Education Statistics, State Public Elementary and Secondary Enrollment Model, 1980–2003; and State Public High School Graduates Model, 1980–81 through 2002–03. (This table was prepared November 2005.)

Table A-14. Equations for full-time and part-time college enrollment rates of White, non-Hispanic, men

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-7.72	0.15	-52.8	0.98	0.74
Age 18	-4.81	0.11	-41.9		
Age 19	-4.61	0.11	-41.8		
Age 20	-4.83	0.11	-43.6		
Age 21	-4.97	0.11	-44.8		
Age 22	-5.47	0.11	-47.9		
Age 23	-5.97	0.11	-53.4		
Age 24	-6.33	0.11	-55.6		
Age 25-29	-7.26	0.11	-65.3		
Age 30-34	-8.34	0.11	-73.5		
Age 35 and up	-8.95	0.12	-74.7		
LNRYPDNWNH	0.22	0.01	38.2		
Part-time					
Age 17	-7.05	0.96	-7.4	0.46	2.12
Age 18	-2.43	0.06	-43.1		
Age 19	-2.15	0.09	-24.6		
Age 20	-2.10	0.05	-42.3		
Age 21	-2.21	0.06	-35.8		
Age 22	-2.10	0.06	-36.6		
Age 23	-2.36	0.04	-53.5		
Age 24	-2.60	0.06	-46.8		
Age 25-29	-2.67	0.04	-74.2		
Age 30-34	-3.12	0.05	-68.3		
Age 35 and up	-3.17	0.03	-107.7		
LNRJECIWSSPCPI	0.67	0.13	5.0		

 $R^2$  = Coefficient of determination.

D.W. statistic = Durbin-Watson statistic.

#### Where:

AGE(age) = Enrollment rate by age.

LNRYPDNWNH = Log of White non-Hispanic per capita disposable income.

LNRJECIWSSPCPI = Log of real total private compensation employment cost index.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

Table A-15. Equations for full-time and part-time college enrollment rates of White, non-Hispanic, women

Independent variable	Coefficient	Standard error	T-statistic	R <sup>2</sup>	D.W. statistic
Full-time					
Age 17	-12.12	0.27	-45.5	0.97	0.56
Age 18	-9.25	0.25	-37.7		
Age 19	-9.17	0.24	-37.5		
Age 20	-9.45	0.24	-38.7		
Age 21	-9.70	0.24	-39.7		
Age 22	-10.50	0.25	-42.5		
Age 23	-10.98	0.25	-44.6		
Age 24	-11.27	0.24	-46.0		
Age 25-29	-12.19	0.24	-49.9		
Age 30-34	-12.89	0.24	-52.7		
Age 35 and up	-13.07	0.24	-53.4		
LNRYPDNWNH	0.47	0.01	37.0		
Part-time Part-time					
Age 17	-8.14	0.42	-19.3	0.80	1.00
Age 18	-4.19	0.23	-18.0		
Age 19	-3.94	0.24	-16.4		
Age 20	-3.93	0.23	-16.9		
Age 21	-4.11	0.24	-17.4		
Age 22	-4.02	0.23	-17.3		
Age 23	-4.32	0.23	-18.6		
Age 24	-4.47	0.23	-19.1		
Age 25-29	-4.60	0.23	-20.0		
Age 30-34	-4.89	0.23	-21.2		
Age 35 and up	-4.59	0.23	-20.0		
LNRYPDNWNH	0.10	0.01	8.7		

 $R^2$  = Coefficient of determination.

#### Where:

AGE(age) = Enrollment rate by age.

LNRYPDNWNH = Log of White non-Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

D.W. statistic = Durbin-Watson statistic.

Table A-16. Equations for full-time and part-time college enrollment rates of Black, non-Hispanic, men

Independent variable	Coefficient	Standard error	T-statistic	R <sup>2</sup>	D.W. statistic
Full-time					
Age 17	-9.13	0.40	-22.6	0.92	1.76
Age 18	-6.96	0.40	-17.6		
Age 19	-6.71	0.40	-17.0		
Age 20	-6.85	0.39	-17.3		
Age 21	-7.06	0.40	-17.8		
Age 22	-7.26	0.40	-18.3		
Age 23	-7.73	0.40	-19.2		
Age 24	-8.00	0.40	-20.1		
Age 25-29	-8.77	0.40	-22.0		
Age 30-34	-9.59	0.41	-23.7		
Age 35 and up	-10.03	0.40	-25.1		
LNRYPDNBNH	0.29	0.02	13.5		
Part-time					
Age 17	-10.22	1.01	-10.1	0.27	2.25
Age 18	-7.74	0.46	-16.9		
Age 19	-7.01	0.45	-15.6		
Age 20	-6.91	0.44	-15.7		
Age 21	-6.93	0.43	-16.1		
Age 22	-6.78	0.45	-15.1		
Age 23	-7.33	0.45	-16.3		
Age 24	-7.22	0.45	-16.2		
Age 25-29	-7.26	0.43	-16.9		
Age 30-34	-7.45	0.43	-17.4		
Age 35 and up	-7.58	0.43	-17.8		
LNRYPDNBNH	0.21	0.02	9.2		

 $R^2$  = Coefficient of determination.

#### Where:

D.W. statistic = Durbin-Watson statistic.

AGE(age) = Enrollment rate by age.

LNRYPDNBNH = Log of Black non-Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions by Race/Ethnicity Model, 1980–2004. (This table was prepared January 2006.)

Table A-17. Equations for full-time and part-time college enrollment rates of Black, non-Hispanic, women

Independent variable	Coefficient	Standard error	T-statistic	R <sup>2</sup>	D.W. statistic
Full-time					
Age 17	-12.37	0.43	-28.9	0.93	1.36
Age 18	-10.34	0.42	-24.8		
Age 19	-10.11	0.42	-24.3		
Age 20	-10.38	0.42	-24.9		
Age 21	-10.49	0.42	-25.2		
Age 22	-11.06	0.42	-26.6		
Age 23	-11.26	0.42	-27.0		
Age 24	-11.51	0.42	-27.5		
Age 25-29	-12.44	0.42	-29.7		
Age 30-34	-12.90	0.42	-31.0		
Age 35 and up	-13.27	0.42	-31.8		
LNRYPDNBNH	0.50	0.02	22.3		
Part-time					
Age 17	-13.39	0.61	-22.0	0.51	1.52
Age 18	-11.15	0.48	-23.4		
Age 19	-10.94	0.47	-23.1		
Age 20	-10.73	0.48	-22.6		
Age 21	-10.86	0.47	-22.9		
Age 22	-10.54	0.47	-22.4		
Age 23	-10.81	0.47	-22.8		
Age 24	-11.19	0.48	-23.5		
Age 25-29	-11.14	0.46	-24.0		
Age 30-34	-11.21	0.47	-24.1		
Age 35 and up	-11.12	0.46	-24.0		
LNRYPDNBNH	0.45	0.03	17.8		

 $R^2$  = Coefficient of determination.

D.W. statistic = Durbin-Watson statistic.

#### Where:

AGE(age) = Enrollment rate by age.

LNRYPDNBNH = Log of Black non-Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

Table A-18. Equations for full-time and part-time college enrollment rates of Hispanic men

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-9.23	0.61	-15.1	0.89	1.98
Age 18	-7.32	0.60	-12.1		
Age 19	-7.08	0.60	-11.7		
Age 20	-7.27	0.60	-12.1		
Age 21	-7.48	0.61	-12.3		
Age 22	-8.03	0.61	-13.3		
Age 23	-8.25	0.61	-13.6		
Age 24	-8.32	0.60	-13.8		
Age 25-29	-9.20	0.61	-15.2		
Age 30-34	-9.94	0.61	-16.4		
Age 35 and up	-10.56	0.61	-17.2		
LNRYPDNH	0.29	0.03	8.7		
Part-time					
Age 17	-9.35	1.05	-8.9	0.26	2.17
Age 18	-6.46	0.50	-13.0		
Age 19	-6.33	0.51	-12.5		
Age 20	-6.09	0.50	-12.2		
Age 21	-6.16	0.50	-12.4		
Age 22	-6.24	0.50	-12.6		
Age 23	-6.54	0.51	-12.8		
Age 24	-6.76	0.50	-13.5		
Age 25-29	-6.82	0.49	-14.0		
Age 30-34	-7.22	0.49	-14.8		
Age 35 and up	-7.27	0.49	-14.9		
LNRYPDNH	0.19	0.03	6.9		

 $R^2$  = Coefficient of determination.

#### Where:

AGE(age) = Enrollment rate by age.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

D.W. statistic = Durbin-Watson statistic.

LNRYPDNH = Log of Hispanic per capita disposable income.

Table A-19. Equations for full-time and part-time college enrollment rates of Hispanic women

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-16.14	0.63	-25.5	0.87	1.78
Age 18	-13.53	0.60	-22.4		
Age 19	-13.47	0.60	-22.4		
Age 20	-13.82	0.60	-22.9		
Age 21	-13.93	0.60	-23.1		
Age 22	-14.65	0.61	-24.1		
Age 23	-14.81	0.60	-24.5		
Age 24	-15.22	0.61	-24.9		
Age 25-29	-15.93	0.60	-26.5		
Age 30-34	-16.60	0.61	-27.4		
Age 35 and up	-16.87	0.61	-27.7		
LNRYPDNH	0.67	0.03	20.2		
Part-time Part-time					
Age 17	-14.30	0.56	-25.4	0.47	1.66
Age 18	-12.03	0.43	-27.7		
Age 19	-11.88	0.43	-27.8		
Age 20	-12.07	0.44	-27.6		
Age 21	-11.94	0.44	-27.4		
Age 22	-12.12	0.44	-27.8		
Age 23	-12.10	0.43	-28.1		
Age 24	-12.52	0.44	-28.6		
Age 25-29	-12.58	0.42	-29.9		
Age 30-34	-12.95	0.42	-30.7		
Age 35 and up	-12.84	0.42	-30.6		
LNRYPDNH	0.52	0.02	22.7		

 $R^2$  = Coefficient of determination.

D.W. statistic = Durbin-Watson statistic.

#### Where:

AGE(age) = Enrollment rate by age.

LNRYPDNH = Log of Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

Table A-20. Equations for full-time and part-time college enrollment rates of Asian/Pacific Islander men

Independent variable	Coefficient	Standard error	T-statistic	$\mathbb{R}^2$	D.W. statistic
Full-time					
Age 17	-7.86	0.71	-11.1	0.92	1.61
Age 18	-4.93	0.69	-7.2		
Age 19	-4.71	0.69	-6.8		
Age 20	-4.86	0.69	-7.1		
Age 21	-4.91	0.69	-7.1		
Age 22	-5.15	0.69	-7.5		
Age 23	-5.38	0.69	-7.8		
Age 24	-5.77	0.69	-8.3		
Age 25-29	-6.58	0.69	-9.5		
Age 30-34	-7.58	0.69	-11.0		
Age 35 and up	-8.35	0.69	-12.1		
LNRYPDNAHNH	0.24	0.04	6.6		
Part-time					
Age 17	-6.51	1.69	-3.9	0.24	2.27
Age 18	-3.46	0.91	-3.8		
Age 19	-2.93	0.92	-3.2		
Age 20	-2.67	0.93	-2.9		
Age 21	-3.00	0.93	-3.2		
Age 22	-2.90	0.95	-3.1		
Age 23	-3.05	0.92	-3.3		
Age 24	-3.56	0.92	-3.9		
Age 25-29	-3.57	0.90	-4.0		
Age 30-34	-4.16	0.90	-4.6		
Age 35 and up	-4.54	0.90	-5.0		
LNRYPDNAHNH	0.07	0.05	1.4		

 $R^2$  = Coefficient of determination.

#### Where:

AGE(age) = Enrollment rate by age.

D.W. statistic = Durbin-Watson statistic.

LNRYPDNAHNH = Log of Asian/Pacific Islander non-Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall, Inc., 1978, pp. 165–173.

SOURCE: U.S. Department of Education, National Center for Education Statistics, Enrollment in Degree-Granting Institutions by Race/Ethnicity Model, 1980–2004. (This table was prepared January 2006.)

Table A-21. Equations for full-time and part-time college enrollment rates of Asian/Pacific Islander women

Independent variable	Coefficient	Standard error	T-statistic	R <sup>2</sup>	D.W. statistic
Full-time					
Age 17	-13.28	0.38	-35.0	0.94	1.44
Age 18	-11.15	0.36	-30.6		
Age 19	-10.53	0.38	-27.9		
Age 20	-10.96	0.37	-29.9		
Age 21	-10.88	0.37	-29.6		
Age 22	-11.47	0.37	-30.6		
Age 23	-11.88	0.37	-32.3		
Age 24	-12.26	0.39	-31.1		
Age 25-29	-13.26	0.36	-36.8		
Age 30-34	-14.60	0.37	-39.5		
Age 35 and up	-14.94	0.37	-40.3		
LNRYPDNAHNH	0.57	0.02	30.4		
Part-time					
Age 17	-17.01	1.00	-17.0	0.47	1.68
Age 18	-14.89	0.85	-17.6		
Age 19	-14.48	0.87	-16.7		
Age 20	-14.90	0.85	-17.5		
Age 21	-14.22	0.86	-16.5		
Age 22	-14.23	0.85	-16.8		
Age 23	-14.94	0.85	-17.6		
Age 24	-15.11	0.86	-17.6		
Age 25-29	-15.50	0.84	-18.5		
Age 30-34	-16.19	0.84	-19.3		
Age 35 and up	-15.98	0.84	-19.1		
LNRYPDNAHNH	0.68	0.04	15.7		

 $R^2$  = Coefficient of determination.

#### Where:

AGE(age) = Enrollment rate by age.

LNRYPDNAHNH = Log of Asian/Pacific Islander non-Hispanic per capita disposable income.

NOTE: The regression method used to estimate the full-time and part-time equations was the pooled seemingly unrelated regression method. The time period used to estimate the equations is from 1980 to 2004. The number of observations is 275. For additional information, see M. D. Intriligator, *Econometric Models, Techniques, & Applications*, New Jersey: Prentice-Hall. Inc. 1978, pp. 165–173.

D.W. statistic = Durbin-Watson statistic.

Table A-22. Enrollment (assumptions)

Variable	Assumptions	Alternatives	Tables
Elementary and secondary enrollment	Age-specific enrollment rates will remain constant at levels consistent with the most recent rates.	Middle (no alternatives)	1-9
	Public enrollment rates and public grade retention rates will remain constant at levels consistent with the most recent rates.	Middle (no alternatives)	1-9
	The percentage of 7th- and 8th-grade public students enrolled in schools organized as secondary schools will remain constant at levels consistent with the most recent rates.	Middle (no alternatives)	1-9
College enrollment, by age, sex, and attendance status	Age-specific enrollment rates are a function of dummy variables by age, middle alternative log of four-period weighted average of real disposable income per capita, and middle alternative log unemployment rate by age group.	Middle	10-19
	Age-specific enrollment rates are a function of dummy variables by age, low alternative log of four-period weighted average of real disposable income per capita, and low alternative log unemployment rate by age group.	Low	10-19
	Age-specific enrollment rates are a function of dummy variables by age, high alternative log of four-period weighted average of real disposable income per capita, and high alternative log unemployment rate by age group.	High	10-19
College enrollment, by sex, attendance status, level enrolled, and type of institution	For each group and for each attendance status separately, percent of total enrollment by sex, level enrolled, and type of institution will follow past trends through 2015. For each age group and attendance status category, the sum of the percentages must equal 100 percent.	High, middle, and low	10-19
College enrollment, by control of institution	For each enrollment category, by sex, attendance status, and level enrolled, and by type of institution, public enrollment as a percent of total enrollment will remain constant at levels consistent with the most recent rates.	High, middle, and low	10-19
Graduate enrollment	For each enrollment category, by sex and attendance status of student, and by type and control of institution, graduate enrollment as a percent of postbaccalaureate enrollment will remain constant at levels consistent with the most recent rates.	and low	20
College enrollment, by age, sex, attendance status, and race/ethnicity			
Full-time: White, Non-Hispanic men; White, Non-Hispanic women; Black, Non-Hispanic men; Black, Non-Hispanic women; Hispanic men; Hispanic women; Asian/Pacific Islander, Non-Hispanic men; Asian/Pacific Islander, Non-Hispanic women. Part-time: White, Non-Hispanic women; Black, Non-Hispanic men; Black, Non-Hispanic women; Hispanic men; Hispanic women; Asian/Pacific Islander, Non-Hispanic men; Asian/Pacific Islander, Non-Hispanic women	Age-specific enrollment rates by race/ethnicity are a function of dummy variables by age and the log of the corresponding race/ethnicity group's real disposable income per capita.	Middle (no alternatives)	22
Part-time White, Non-Hispanic men	Age-specific enrollment rates by race/ethnicity are a function of dummy variables by age and the log of real total compensation.		22
Full-time-equivalent of part-time enrollment	For each enrollment category, by type and control of institution and level enrolled, the percent that full-time-equivalent of part-time enrollment is of part-time enrollment will remain constant at levels consistent with the most recent rates.	High, middle, and low	23

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Elementary and Secondary Enrollment Model, 1972–2004; State Public Elementary and Secondary Enrollment Model, 1980–2004; Enrollment in Degree-Granting Institutions Model, 1980–2004; and Enrollment in Degree-Granting Institutions by Race/Ethnicity Model, 1980–2004. (This table was prepared November 2005.)

## High School Graduates

#### **National**

Projections of public high school graduates were developed in the following manner. The number of public high school graduates was expressed as a percent of grade 12 enrollment in public schools for 1972-73 to 2002-03. This percent was projected using single exponential smoothing and applied to projections of grade 12 enrollment to yield projections of high school graduates in public schools. (This percent does not make any specific assumptions regarding the dropout rate. The effect of the 12th- grade dropout proportion is reflected implicitly in the graduate proportion.) The grade 12 enrollment was projected based on grade progression rates. This percent was assumed to remain constant at levels consistent with the most recent rates. This method assumes that past trends in factors affecting graduation ratios, such as dropouts, migration, and public or private transfers, will continue over the projection period. In addition to student behaviors, the projected number of graduates could be affected by changes in graduation requirements.

Projections of private high school graduates were calculated using the same methodology as public high school graduates, using data from 1988–89 to 2000–01.

#### **Projection Accuracy**

An analysis of projections from models used in the past 15 editions of *Projections of Education Statistics* indicates that the mean absolute percentage errors (MAPEs) for projections of public high school graduates were 0.8 percent for 1 year ahead, 0.9 percent for 2 years

ahead, 1.4 percent for 5 years ahead, and 3.9 percent for 10 years ahead. For the 1-year-ahead prediction, this meansthat one would expect the projection to be within 0.8 percent of the actual value, on the average. For more information on the mean absolute percentage errors, see table A-2, page 89.

#### State Level

This edition contains projections of high school graduates from public schools by state from 2003–04 to 2015–16. Public school graduate data from the Common Core of Data survey for 1980–81 to 2002–03 were used to develop these projections. This survey does not collect graduate data for private schools.

Projections of public high school graduates by state were developed in the following manner. For each state, the number of public high school graduates was expressed as a percent of grade 12 enrollment in public schools for 1980-81 to 2002-03. This percent was projected using single exponential smoothing and applied to projections of grade 12 enrollment to yield projections of high school graduates in public schools. All states, with the exception of the District of Columbia, were projected using the same single exponential smoothing parameter. Due to questions about the quality of the District of Columbia data, the smoothing parameters for the District of Columbia were estimated using the available historical data. This approach yielded more consistent projections of the District of Columbia graduates. Projections of grade 12 enrollment were developed based on the grade progression rates discussed in appendix A, Enrollment. The projected rates were assumed to remain constant at levels consistent with the most recent rates. This method assumes that past trends in factors affecting public high school graduates will continue over the projection period.

## **Degrees Conferred**

Projections of associate's, bachelor's, master's, doctor's, and first-professional degrees for men and women were based on demographic models that relate degree awards to college-age populations and college enrollment by level enrolled and attendance status. Table A-23 describes the equations used to calculate projections, and table A-24 contains the basic assumptions underlying projections.

#### Associate's Degrees

Associate's degree projections for men and women were based on a weighted average over the last 2 years of undergraduate enrollment by attendance status in 2-year institutions and sex relative to the 18- to 24-year-old population by sex. The previous year is weighted two-thirds, and 2 years back is weighted one-third. Results of the regression analysis used to project associate's degrees are shown in table A-23.

#### **Bachelor's Degrees**

Bachelor's degree projections for men and women were based on a weighted average over the last 4 years of undergraduate enrollment by attendance status in 4-year institutions and sex relative to the 18- to 24-year-old population by sex. The weights for the previous 4 years—0.4, 0.3, 0.2, and 0.1—give more weight to the most recent years. Results of the regression analysis used to project bachelor's degrees are shown in table A-23.

#### Master's Degrees

Master's degree projections for men and women were based on a weighted average over the last 2 years of graduate enrollment by attendance status and sex relative to the 25- to 34-year-old population by sex. The previous year is weighted two-thirds, and 2 years back is weighted one-third. Results of the regression analysis used to project master's degrees are shown in table A-23.

#### **Doctor's Degrees**

Doctor's degree projections for men and women were based on a weighted average over the last 4 years of graduate enrollment by attendance status and sex relative to the 35- to 44-year-old population, by sex. The weights for the previous 4 years—0.4, 0.3, 0.2, and 0.1—give more weight to the most recent years. The results of the regression analysis used to project doctor's degrees are shown in table A-23.

#### First-Professional Degrees

First-professional degree projections for men and women were based on a weighted average over the last 3 years of first-professional enrollment by attendance status in 4-year institutions and sex relative to the 25- to 34-year-old population by sex. The weights for the previous 3 years—0.5, 0.33, and 0.17—give more weight to the most recent years. Results of the regression analysis used to project first-professional degree are shown in table A-23.

#### **Projection Accuracy**

An analysis of projection errors from similar models used in the past nine editions of Projections of Education Statistics indicates that mean absolute percentage errors (MAPEs) for associate's degrees were 2.3 percent for 1 year out, 2.9 percent for 2 years out, and 5.6 percent for 5 years out. For the 1-year-out prediction, this means that one would expect the projection to be within 2.3 percent of the actual value, on average. MAPEs for bachelor's degree projections were 0.9 percent for 1 year out, 2.0 percent for 2 years out, and 5.7 percent for 5 years out. MAPEs for master's degrees were 1.6, 4.1, and 11.3 percent, respectively. For doctor's degrees, the MAPEs were 2.6, 3.5, and 2.5 percent, respectively. For first-professional degrees, the MAPEs were 1.3, 1.3, and 5.5 percent, respectively. For more information on the MAPEs, see table A-2.

Table A-23. Equations for degrees conferred

-							Durbin-Watson	Error distribution		Time
Dependent variable					Equation	R <sup>2</sup>	statistic <sup>1</sup>	pattern <sup>2</sup>	Rho	period
Associate's degrees Men	LNASSOCM	=	4.9	+	0.4LNUG2ML2 (10.2)	0.95	2.0	AR(1)	0.57 (5.1)	1975–76 to 2003–04
Associate's degrees Women	LNASSOCW	=	5.6	+	0.6LNUG2WL2 (17.2)	0.99	1.5	AR(1)	0.65 (8.3)	1975–76 to 2003–04
Bachelor's degrees Men	LNBACHM	=	6.3	-	0.5LNUG4ML4 (5.2)	0.98	1.5	AR(1)	0.97 (7.5)	1977–78 to 2003–04
Bachelor's degrees Women	LNBACHW	=	139.6	-	0.4LNUG4WL4 (4.6)	0.99	1.6	AR(1)	0.99 (22.4)	1977–78 to 2003–04
Master's degrees Men	LNMASTM	=	7.0	+	0.6LNGML2 (4.6)	0.97	1.4	AR(1)	1.02 (9.6)	1975–76 to 2003–04
Master's degrees Women	LNMASTW	=	7.7	+	0.7LNGWL2 (22.3)	0.99	1.4	AR(1)	0.79 (18.6)	1975–76 to 2003–04
Doctor's degrees Men	LNDOCM	=	3.3	+	0.4LNGML4 (6.8)	0.99	1.7	AR(1)	0.69 (7.1)	1977–78 to 2003–04
Doctor's degrees Women	LNDOCW	=	1.9	+	0.3LNGWL4 (3.2)	0.95	2.3	AR(1)	1.04 (17.1)	1977–78 to 2003–04
First-professional degrees Men	LNFPROM	=	3.5	+	0.2LNFPML3 (3.1)	0.99	1.7	AR(1)	0.87 (23.1)	1976–77 to 2003–04
First-professional degrees Women	LNFPROW	=	7.8	+	0.6LNFPWL3 (35.6)	0.98	0.8	AR(1)	0.07 (0.6)	1976–77 to 2003–04

For an explanation of the Durbin-Watson statistic, see J. Johnston and J. Dinardo, Econometric Methods, New York: McGraw-Hill, 1996.

<sup>&</sup>lt;sup>2</sup>AR(1) indicates that the models was estimated using least squares with the AR(1) process for correcting for first-order autocorrelation. For a general discussion of the problem of autocorrelation, and the method used to forecast in the presence of autocorrelation, see G. Judge, W. Hill, R. Griffiths, H. Lutkepohl, and T. Lee, *The Theory and Practice of Econometrics*, New York: John Wiley and Sons, 1985, pp. 315–318.

Where-

LNASSOCM = Log of the ratio of associate's degrees awarded to men relative to the population of 18- to 24-year old men

LNASSOCW = Log of the ratio of associate's degrees awarded to woman relative to the population of 18- to 24-year old women

LNBACHM = Log of the ratio of bachelor's degrees awarded to men relative to the population of 18- to 24-year old men

LNBACHW = Log of the ratio of bachelor's degrees awarded to women relative to the population of 18- to 24-year old women

LNMASTM = Log of the ratio of master's degrees awarded to men relative to the population of 25- to 34-year old men

LNMASTW = Log of the ratio of master's degrees awarded to women relative to the population of 25- to 34-year old women LNDOCM = Log of the ratio of doctor's degrees awarded to men relative to the population of 35- to 44-year old men

LNDOCW = Log of the ratio of doctor's degrees awarded to women relative to the population of 35- to 44-year old women

LNFPROM = Log of the ratio of doctor's degrees awarded to women relative to the population of 35- to 44-year old women LNFPROM = Log of the ratio of first-professional degrees awarded to men relative to the population of 25- to 34-year old men

LNFPROW = Log of the ratio of first-professional degrees awarded to women relative to the population of 25- to 34-year old women

LNUG2ML2 = Log of the ratio of full-time male undergraduate enrollment in 2-year institutions to the male population of 18- to 24-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time male undergraduate enrollment in 2-year institutions.

LNUG2WL2 = Log of the ratio of full-time female undergraduate enrollment in 2-year institutions to the female population of 18- to 24-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time female undergraduate enrollment in 2-year institutions.

LNUG4ML4 = Log of the ratio of full-time male undergraduate enrollment in 4-year institutions to the male population of 18- to 24-year-olds, weighted over the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time male undergraduate enrollment in 4-year institutions.

LNUG4WL4 = Log of the ratio of full-time female undergraduate enrollment in 4-year institutions to the female population of 18- to 24-year-olds, weighted over the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time female undergraduate enrollment in 4-year institutions.

LNGML2 = Log of the ratio of full-time male graduate enrollment to the male population of 25- to 34-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time male graduate enrollment.

LNGWL2 = Log of the ratio of full-time female graduate enrollment to the female population of 25- to 34-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time female graduate enrollment.

LNGML4 = Log of the ratio of full-time male graduate enrollment to the male population of 35- to 44-year-olds, weighted over the last 4 years (where weights are .4, .3., .2, and .1 for descending lagged years), plus the similar log ratio for part-time male graduate enrollment.

LNGWL4 = Log of the ratio of full-time female graduate enrollment to the female population of 35- to 44-year-olds, weighted over the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time female graduate enrollment.

LNFPML3 = Log of the ratio of full-time male first-professional enrollment to the male population of 25- to 34-year-olds, weighted over the last 3 years (where weights are .5, .33, and .17 for descending lagged years), plus the similar log ratio for part-time male first-professional enrollment.

LNFPWL3 = Log of the ratio of full-time female first-professional enrollment to the female population of 25- to 34-year-olds, weighted over the last 3 years (where weights are .5, .33, and .17 for descending lagged years), plus the similar log ratio for part-time female first-professional enrollment.

NOTE: R<sup>2</sup> indicates coefficient of determination. Rho measures the correlation between errors in time period t and time period t minus 1. Numbers in parentheses are t-statistics. SOURCE: U.S. Department of Education, National Center for Education Statistics, Degrees Conferred Model, 1975–76 through 2003–04. (This table was prepared November 2005.)

Table A-24. Degrees conferred (assumptions)

Variable	Assumptions	Alternatives	Tables	
Associate's degrees				
Men	The number of associate's degrees awarded to men is a linear function of the log of the ratio of full-time male undergraduate enrollment in 2-year institutions to the male population of 18- to 24-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time male undergraduate enrollment in 2-year institutions. This relationship will continue through 2015–16.	Middle	27	
Women	The number of associate's degrees awarded to women is a linear function of the log of the ratio of full-time female undergraduate enrollment in 2-year institutions to the female population of 18- to 24-year-olds, weighted over the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time female undergraduate enrollment in 2-year institutions. This relationship will continue through 2015–16.	Middle	27	
Bachelor's degrees				
Men	The number of bachelor's degrees awarded to men is a linear function of the log of the ratio of full-time male undergraduate enrollment in 4-year institutions to the male population of 18- to 24-year-olds, weighted over the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time male undergraduate enrollment in 4-year institutions. This relationship will continue through 2015–16.	Middle	28	
Women	The number of bachelor's degrees awarded to women is a linear function of the log of the ratio of full-time female undergraduate enrollment in 4-year institutions to the female population of 18- to 24-year-olds, weighted over the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time female undergraduate enrollment in 4-year institutions.  This relationship will continue through 2015–16.	Middle	28	
Master's degrees				
Men	The number of master's degrees awarded to men is a linear function of the log of the ratio of full-time male graduate school enrollment to the male population of 25- to 34-year-olds, weighted over the the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time male graduate school enrollment. This relationship will continue through 2015–16.	Middle	29	
Women	The number of master's degrees awarded to women is a linear function of the log of the ratio of full-time female graduate school enrollment to the female population of 25- to 34-year-olds, weighted over the the last 2 years (where weights are .67 and .33 for descending lagged years), plus the similar log ratio for part-time female graduate school enrollment. This relationship will continue through 2015–16.	Middle	29	
Doctor's degrees				
Men	The number of doctor's degrees awarded to men is a linear function of the log of the ratio of full-time male graduate school enrollment to the male population of 35- to 44-year-olds, weighted over the the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time male graduate school enrollment. This relationship will continue through 2015–16.	Middle	30	
Women	The number of doctor's degrees awarded to women is a linear function of the log of the ratio of full-time female graduate school enrollment to the female population of 35- to 44-year-olds, weighted over the the last 4 years (where weights are .4, .3, .2, and .1 for descending lagged years), plus the similar log ratio for part-time female graduate school enrollment. This relationship will continue through 2015–16.	Middle	30	
First-professional degrees				
Men	The number of associate's degrees awarded to men is a linear function of the log of the ratio of fi male undergraduate enrollment in 2-year institutions to the male population of 18 he to 24-year weighteed over the last 22 years (where weights are, 6.7 and, 35 for descending lagged years), psimilar log ratio for part-time male undergraduate enrollment in 2-year institutions. This relative the population of 18 will continue through 20 of the full-time female undergraduate enrollment in 2-year institutions to the female population of 18 year-olds, weighted over the last 2 years (where weights are, 6.7 and, 35 for descending lagged plus the similar log ratio for part-time female undergraduate enrollment in 2-year institutions to the female population of 18 to 24-year descending lagged years), psimilar log ratio for part-time female undergraduate enrollment in 2-year institutions or the male population of 18 to 24-year weighted over the last 2 years (where weights are, 6.3, 2, and 1.1 for descending lagged years), similar log ratio for part-time male undergraduate enrollment in 4-year institutions. This relatively weighted over the last 2 years (where weights are, 6.3, 2, and 1.1 for descending lagged years), psimilar log ratio for part-time male undergraduate enrollment in 4-year institutions to the female population of 24-year-olds, weighted over the last 4 years (where weights are, 6.3, 2, and 1.1 for descending years), plus the similar log ratio for part-time female undergraduate enrollment in 4-year institutions to the female population of 24-year-olds, weighted over the last 4 years (where weights are, 6.3, 2, and 1.1 for descending lagged years), plus the similar log ratio for part-time female undergraduate enrollment in 4-year institutions to the female population of 25 to 34-year-olds, weighted over the last 4 years (where weights are 6.7 and .33 for descending lagged years), plus the similar log artio for part-time female graduate school enrollment. This relationship will continue through 20 time female g			
Women	The number of first-professional degrees awarded to women is a linear function of the log of the ratio of full-time female first-professional school enrollment to the female population of 25- to 34-year-olds, weighted over the last 3 years (where weights are .5, .33, and, .17 for descending lagged years), plus the similar log ratio for part-time female first-professional school enrollment. This relationship will continue through 2015–16.	Middle	31	

SOURCE: U.S. Department of Education, National Center for Education Statistics, Degrees Conferred Model, 1975–76 through 2003–04. (This table was prepared November 2005.)

# Elementary and Secondary Teachers

#### **Public Elementary and Secondary Teachers**

The number of public elementary and secondary teachers was projected separately for the elementary and secondary levels. The number of public elementary teachers was projected using the public elementary student/teacher ratio. The ratio was modeled as a function of local education revenue from state sources per student, and the level of elementary and secondary teacher wages relative to the overall economy-level wages. The number of public elementary teachers was obtained by applying the projected public elementary student/teacher ratio to the previously projected enrollment in public elementary schools. The number of public secondary teachers was projected using the public secondary student/teacher ratio. The ratio was modeled as a function of local education revenue from state sources per student and public secondary enrollment relative to the 11- to 18year-old population. The number of public secondary teachers was obtained by applying the projected public secondary student/teacher ratio to the previously projected enrollment in public secondary schools.

The models were estimated using the AR1 model for correcting for autocorrelation, and all variables are in log form. Local education revenue from state sources were in constant 2000 dollars.

The equations in this section should be viewed as forecasting rather than structural equations, as the limitations of time and available data precluded the building of a large-scale, structural teacher model. The particular equations shown were selected on the basis of their statistical properties, such as coefficients of determination (R<sup>2</sup>s), the t-statistics of the coefficients, the Durbin-Watson statistic, and residual plots.

The multiple regression technique will yield good forecasting results only if the relationships that existed among the variables in the past continue throughout the projection period.

The public elementary teacher model is:

$$ln(RELENRTCH_{t}) = b_{0} + b_{1}ln(RSALARY_{t}) + b_{2}ln(RSGRNTELENR_{t})$$

#### where:

RELENRTCH, is the public elementary student/teacher ratio in year t;

RSALARY<sub>t</sub> is the average teacher wage relative to the overall economy-level wage in year t; and

RSGRNTELENR, is the level of education revenue from state sources deflated by the consumer prices chained-price index in constant 2000 dollars per public elementary student in year t.

Each variable affects the public elementary student/ teacher ratio in the expected way. As the average teacher wage relative to the overall economy-level wage increases, schools economize on teachers by increasing the student/ teacher ratio as teachers are now more expensive to hire. As the level of real grants per elementary student increases, the class size decreases. The more money being devoted to education, the more teachers are hired, thus decreasing the student/teacher ratio.

The public secondary teacher model is:

$$ln(RSCENRTCH_{t}) = b_{0} + b_{1}ln(RSGRNTSCENR_{t}) + b_{2}ln(RSCENRPU_{t})$$

#### where:

RSCENRTCH<sub>t</sub> is the public secondary student/teacher ratio in year t;

RSGRNTSCENR<sub>t</sub> is the level of education revenue from state sources deflated by the consumer prices chained-price index in constant 2000 dollars per public secondary student in year t; and

RSCENRPU<sub>t</sub> is the number of students enrolled in public secondary schools relative to the secondary schoolage population in year t.

Each variable affects the public secondary student/teacher ratio in the expected way. As the level of real grants per secondary student increases, the student/teacher ratio decreases. The more money being devoted to education, the more teachers are hired, thus decreasing the student/teacher ratio. As enrollment rates (number of enrolled students relative to the school-age population) increase, the ratio also increases: increases in the enrollment rate are not matched one-for-one in increases in the number of teachers.

Table A-25 summarizes the results for the elementary and secondary public teacher models.

Enrollment is by organizational level, not by grade level. Thus, secondary enrollment is not the same as grade 9–12 enrollment because some states count some grade 7 and 8 enrollment as secondary. Therefore, the distribution of the number of teachers is also by organizational level, not by grade span.

#### **Private Elementary and Secondary Teachers**

Projections of private elementary and secondary teachers were derived in the following manner. From 1960 to 2002, the ratio of private school teachers to public school teachers was calculated by organizational level. These ratios were projected using single exponential smoothing, yielding a constant value over the projection period. This constant value was then applied to projections of public school teachers by organizational level to yield projections

of private school teachers. This method assumes that the future pattern in the trend of private school teachers will be the same as that for public school teachers. The reader is cautioned that a number of factors could alter the assumption of constant ratios over the projection period.

The total number of public school teachers, enrollment by organizational level, and education revenue from state sources used in these projections were from the Common Core of Data (CCD) survey conducted by NCES. The proportion of public school teachers by organizational level was taken from the National Education Association and then applied to the total number of teachers from the CCD to produce the number of teachers by organizational level.

#### **Projection Accuracy**

An analysis of projection errors from the past 15 editions of *Projections of Education Statistics* indicated that the mean absolute percentage errors (MAPEs) for projections of classroom teachers in public elementary and secondary schools were 1.0 percent for 1 year out, 1.6 percent for 2 years out, 2.7 percent for 5 years out, and 5.6 percent for 10 years out. For the 2-year-ahead prediction, this means that one would expect the projection to be within 1.6 percent of the actual value, on average. For more information on the MAPEs, see table A-2.

Table A-25. Equations for public elementary and secondary teachers

Dependent variable						Equation	$\mathbb{R}^2$	Durbin- Watson statistic <sup>1</sup>	Error distribution pattern <sup>2</sup>	Rho	Time period
Elementary	ln(RELENRTCH)	=	3.8	.1 ln(RSALARY) (4.8)	-	2 ln(RSGRNTELENR) (-8.5)	0.99	1.9	AR(1)	0.21 (1.24)	1968 to 2002
Secondary	ln(RSCENRTCH)	=	4.1	.2 ln(RSGRNTSCENR) (-13.8)	+	.6 ln(RSCENRPU) (4.7)	0.99	1.9	AR(1)	0.60 (3.6)	1973 to 2002

<sup>&</sup>lt;sup>1</sup>For an explanation of the Durbin-Watson statistic, see J. Johnston and J. Dinardo, Econometric Methods, New York: McGraw-Hill, 1996.

#### W/here

RELENRTCH = Log of the ratio of public elementary school enrollment to classroom teachers (i.e., student/teacher ratio)

RSCENRTCH = Log of the ratio of public secondary school enrollment to classroom teachers (i.e., student/teacher ratio)

RSALARY = Log of the average annual teacher salary relative to the overall economy wage in 2000 dollars

RSGRNTELENR = Log of the ratio of education revenue receipts from state sources per capita to public elementary school enrollment in 2000 dollars

RSGRNTSCENR = Log of the ratio of education revenue receipts from state sources per capita to public secondary school enrollment in 2000 dollars

RSCENRPU = Log of the ratio of enrollment in public secondary schools to the 11- to 18-year-old population

NOTE: R<sup>2</sup> indicates the coefficient of determination. Rho measures the correlation between errors in time period t and time period t minus 1. Numbers in parentheses are t-statistics. SOURCE: U.S. Department of Education, National Center for Education Statistics, Elementary and Secondary Teacher Model, 1968–2003. (This table was prepared November 2005.)

<sup>&</sup>lt;sup>2</sup>AR(1) indicates that the models was estimated using least squares with the AR(1) process for correcting for first-order autocorrelation. For a general discussion of the problem of autocorrelation, and the method used to forecast in the presence of autocorrelation, see G. Judge, W. Hill, R. Griffiths, H. Lutkepohl, and T. Lee, *The Theory and Practice of Econometrics*, New York: John Wiley and Sons, 1985, pp. 315–318.

# Expenditures of Public Elementary and Secondary Schools

Econometric techniques were used to produce the projections for current expenditures and average teacher salaries. The particular equations shown were selected on the basis of their statistical properties, such as coefficients of determination (R<sup>2</sup>s), the t-statistics of the variables, the Durbin-Watson statistic, and residual plots. These econometric models will yield good forecasting results only if the relationships that existed among the variables in the past continue throughout the projection period.

#### Elementary and Secondary School Current Expenditure Model

There is a large body of work, both theoretical and empirical, on the demand for local public services such as education.<sup>1</sup> The elementary and secondary school current expenditure model is based on this work.

The model that is the basis for the elementary and secondary school current expenditure model has been called the median voter model. In brief, the theory states that spending for each public good in the community (in this case, spending for education) reflects the preferences of the "median voter" in the community. This individual is identified as the voter in the community with the median income and median property value. The amount of spending in the community reflects the price of education facing the voter with the median income, as well as his income and tastes. There are competing models in which the level of spending reflects the choices of others in the community, such as the "bureaucrats."

In a median voter model, the demand for education expenditures is typically linked to four different types

of variables: (1) measures of the income of the median voter; (2) measures of intergovernmental aid for education going indirectly to the median voter; (3) measures of the price to the median voter of providing one more dollar of education expenditures per pupil; and (4) any other variables that may affect one's tastes for education. The elementary and secondary school current expenditure model contains variables reflecting the first two types of variables. The model is:

 $ln(CUREXP_{r}) = b_{0} + b_{1}ln(PCI_{r}) + b_{2}ln(SGRNT_{r})$ 

#### where:

In indicates the natural log;

CUREXP<sub>t</sub> equals current expenditures of public elementary and secondary schools per pupil in fall enrollment in constant 1982–84 dollars in year t;

PCI<sub>t</sub> equals disposable income per capita in constant 2000 dollars in year t; and

SGRNT<sub>t</sub> equals local governments' education revenue from state sources, per capita, in constant year 1982–84 dollars in year t. The model used to project this variable is discussed below.

The model was estimated using least squares with the AR(1) process for correcting for autocorrelation. This is the 12th edition of *Projections of Education Statistics* in which AR(1) was used. No correction for autocorrelation had been made in the previous four editions of *Projections of Education Statistics*. The model was estimated using data from 1969–70 to 2002–03.

There are potential problems with using a model for local government education expenditures for the nation as a whole. Two such problems concern the variable SGRNT. First, the amount of money that local governments receive for education from state governments varies substantially by state. Second, the formulas used to apportion state moneys for education among local governments vary by state.

Beginning in 1988–89, there was a major change in the survey form used to collect data on current expenditures. This new survey form produces a more complete measure of current expenditures; therefore, the values for current expenditures are not completely comparable to the previously collected numbers. Data for a majority of states were also collected for 1986–87 and 1987–88 that were comparable to data from the new survey

<sup>&</sup>lt;sup>1</sup> For a discussion of the theory together with a review of some of the older literature, see Inman, R. P. (1979), "The Fiscal Performance of Local Governments: An Interpretive Review," in *Current Issues in Urban Economics*, edited by P. Mieszkowski and M. Straszheim, Johns Hopkins Press, Baltimore, Maryland. More recent empirical work includes: Gamkhar, S. and Oates, W. (1996). Asymmetries in the Response to Increases and Decreases in Intergovernmental Grants: Some Empirical Findings. *National Tax Journal*, 49(3): 501-512 and Mitias, P. and Turnbull, G. (2001) Grant Illusion, Tax Illusion, and Local Government Spending. *Public Finance Review*. 29(5): 347-368.

form. A comparison of these data with those from the old survey form suggests that the use of the new survey form may have increased the national figure for current expenditures by approximately 1.4 percent over what it would have been if the survey form had not been changed. When the model was estimated, all values for current expenditures before 1988–89 were increased by 1.4 percent.

The results for the model are shown in table A-26. Each variable affects current expenditures in the direction that would be expected. With high levels of income (PCI) or revenue from state sources (SGRNT), the level of spending increases.

From the cross-sectional studies of the demand for education expenditures, we have an estimate of how sensitive current expenditures are to changes in PCI. We can compare the results from this model with those from the cross-sectional studies. For this model, an increase in PCI of 1 percent, with SGRNT held constant, would result in an increase of current expenditures per pupil in fall enrollment of approximately .77 percent. With PCI held constant, an increase of 1 percent in SGRNT would result in an increase in current expenditures per pupil in fall enrollment of approximately .24 percent. Both numbers are well within the range of what has been found in cross-sectional studies.

The results from this model are not completely comparable with those from previous editions of Projections of Education Statistics. First, in those earlier editions, the sample period used to estimate the model began with either 1959-60 or 1967-68 rather than 1969-70. This change was made due to superior model diagnostics. Second, in some earlier editions the model contained an additional variable, as a proxy for the price facing the median voter, the ratio of enrollment to the population. This price variable has been excluded due to its lack of statistical significance as measured by its t-statistic. Third, in editions prior to Projections of Education Statistics to 2011 and Projections of Education Statistics to 2013,2 average daily attendance rather than fall enrollment, was used as the measure of enrollment. This change was made because the definitions of fall enrollment are more consistent from state to state than those of average daily attendance.

There have been other changes to the model used in earlier editions. As with the current expenditure

projections in the most recent editions, the population number for each school year is the U.S. Census Bureau's July 1 population number for the upcoming school year. In earlier editions, the school year population numbers were from an economic consulting firm. These changes were made to be consistent with population projections used in producing other projections of education statistics. Also, there have been changes in the definition of disposable income.

Projections for total current expenditures were made by multiplying the projections for current expenditures per pupil in fall enrollment by projections for fall enrollment. The projections for total current expenditures were also divided by projections for average daily attendance to produce projections of current expenditures per pupil in average daily attendance to provide projections that are consistent with those from earlier years. Projections were developed in 1982-84 dollars and then placed in 2003-04 dollars using the Consumer Price Index. Current-dollar projections were produced by multiplying the constant-dollar projections by projections for the Consumer Price Index. The Consumer Price Index and the other economic variables used in calculating the projections presented in this report were placed in school year terms rather than calendar year terms.

Three alternative sets of projections for current expenditures are presented: the middle alternative projections, the low alternative projections, and the high alternative projections. The alternative sets of projections differ because of varying assumptions about the growth paths for disposable income and revenue from state sources.

The alternative sets of projections for the economic variables, including disposable income, were developed using three economic scenarios prepared by the economic consulting firm, Global Insight, Inc.

Global Insight's February 2005 trend scenario was used as a base for the middle alternative projections of the economic variables. Global Insight's trend scenario depicts a mean of possible paths that the economy could take over the forecast period, barring major shocks. The economy, in this scenario, evolves smoothly, without major fluctuations.

Global Insight's February 2005 pessimistic scenario was used for the low alternative projections, and Global Insight's February 2005 optimistic scenario was used for the high alternative projections.

<sup>&</sup>lt;sup>2</sup> There were no projections of either current expenditures or teacher salaries in *Projections of Education Statistics to 2012.* 

In the middle alternative projections, disposable income per capita rises each year from 2005–06 to 2015–16 at rates between 2.0 percent and 2.4 percent. In the low alternative projections, disposable income per capita ranges between 1.5 percent and 2.5 percent, and in the high alternative projections, disposable income per capita rises at rates between 2.3 percent and 3.2 percent.

The alternative projections for revenue from state sources, which form a component of the current expenditures model, were produced using the following model:

 $ln(SGRNT_1) = b_0 + b_1 ln(PCI_1) + b_2 ln(ENRPOP_1)$ 

#### where:

In indicates the natural log;

SGRNT<sub>t</sub> equals local governments' education revenue from state sources, per capita, in constant 1982–84 dollars in year t;

ENRPOP<sub>t</sub> equals the ratio of fall enrollment to the population in year t; and

PCI<sub>t</sub> equals disposable income per capita in constant 2000 dollars in year t.

The model was estimated using least squares with the AR(1) process for correcting for autocorrelation. The model was estimated using the period from 1971–72 to 2002–03. These models are shown in table A-26.

The values of the coefficients in this model follow expectations. As the enrollment increases relative to the population (higher ENRPOP), so does the amount of aid going to education. Finally, other things being equal, as the value of disposable income per capita in real dollar values (higher PCI) increases, the level of local governments' education revenue from state sources per capita also increases.

This year's edition of the *Projections of Education Statistics* uses the same revenue from state sources model as last year's edition. The model used in the prior two editions, *Projections of Education Statistics 2012* and *Projections of Education Statistics 2013*, was different. It included a term for personal taxes and non-tax receipts (PERTAX1) and an inflation rate term (RCPIANN) and was estimated over a different time period (the sample period began in 1967-68 rather than 1971-72). As with last year, the current model

specification yielded superior model diagnostics than the model used in the Projections of Education Statistics 2012 and Projections of Education Statistics 2013. The models in the previous four editions of the Projections of Education Statistics each used the same variable to represent enrollment (ENRPOP). In the earlier editions, models used average daily attendance rather than fall enrollment as the measure of enrollment, and the sample period used to produce the forecast began in 1959-60. As with the current expenditures model, the change to fall enrollment was done because the definition of fall enrollment is more consistent across states, and the change in sample period was done because of superior model diagnostics. Other models in the past have contained a second measure of state and local government revenue. Also in earlier editions, similar models were used except the variables were not in log form. Both of these changes were made because of superior model diagnostics.

Three alternative sets of projections for SGRNT were produced using this model. Each is based on a different set of projections for disposable income per capita. The middle set of projections was produced using the values from the middle set of alternative projections. The low set of projections was produced using the values from the low set of alternative projections, and the high set of projections was produced using the values from the high set of alternative projections. In the middle alternative projections, disposable income per capita rises each year from 2005-06 to 2015-16 at rates between 2.0 percent and 2.4 percent. In the low alternative projections, disposable income per capita ranges between 1.5 percent and 2.5 percent, and in the high alternative projections, disposable income per capita rises at rates between 2.3 percent and 3.2 percent.

## Elementary and Secondary Teacher Salary Model

Most studies conducted on teacher salaries, like those on current expenditures, have used cross-sectional data. Unlike current expenditures models, however, the models for teacher salaries from these existing cross-sectional studies cannot easily be reformulated for use with time series data. One problem is that we do not have sufficient information concerning the supply of qualified teachers who are not presently teaching. Instead, the elementary and secondary salary model contains terms that measure the demand for teachers in the economy.

The elementary and secondary teacher salary model is:

$$ln(SALRY_t) = b_0 + b_1 ln(CUREXP_t) + b_2 ln(ENRPOP_t) + b_3 ln(ENR_t/ENR_t)$$

#### where:

In indicates the natural log;

SALRY<sub>t</sub> equals the estimated average annual salary of teachers in public elementary and secondary schools in constant 1982–84 dollars in year t;

CUREXP<sub>t</sub> equals current expenditures of public elementary and secondary schools per pupil in fall enrollment in constant 1982–84 dollars in year t;

ENRPOP<sub>t</sub> equals the ratio of fall enrollment to the population in year t;

ENR equals fall enrollment in year t; and

ENR1 equals fall enrollment in year t-1.

The model was estimated using the period from 1970–71 to 2002–03. The model was estimated using least squares with the AR(1) process for correcting for autocorrelation.

Due to the effects on current expenditures caused by the change in survey forms discussed above, the values for current expenditures for 1969–70 to 1987–88 were increased by 1.4 percent when the salary model was estimated.

The equations and results for this model are also shown in table A-26. There is no literature for comparing the sizes of the coefficients. However, the direction of the impact each variable has on salaries is as expected: as the level of spending per pupil increases (higher CUREXP), more teachers can be hired, so demand for teachers increases and salaries may increase; as the number of students increases (higher ENRPOP and ENR/ENR1), demand for teachers may increase, so salaries may increase.

This year's edition of the *Projections of Education Statistics* uses the same salary model as last year's edition. The model used in the prior two editions, *Projections of Education Statistics 2012* and *Projections of Education Statistics 2013*, was slightly different: the enrollment ratio variable was the ratio of enrollment lagged one period to enrollment lagged two periods. Earlier versions of the salary model used average daily attendance rather

than fall enrollment as the measure of enrollment, and the sample period used to produce the forecast began in 1959–60 rather than 1969–70. As with the current expenditures model, the change to fall enrollment was done because the definition of fall enrollment is more consistent across states.

Beginning with the *Projections of Education Statistics to 2006*, variables were in log form. In earlier editions, they were not.

As with current expenditures, three different scenarios are presented for teacher salaries. The same projections for ENRPOP and ENR are used for each alternative projection; the sole difference between the projections is in the projection for current expenditures. The middle alternative projection for salaries uses the middle alternative projection for current expenditures. The low alternative projection for salaries uses the low alternative projection for current expenditures. The high alternative projection for salaries uses the high alternative projection for current expenditures.

Current expenditures, average teacher salaries, and the number of teachers are interrelated; analysis was conducted to see whether the projections of these three time series were consistent.

The number of teachers was multiplied by the average salary and then divided by current expenditures for every school year from 1987–88 until 2015–16 (using the middle alternative projection for teachers, salaries, and current expenditures). The resulting value shows the portion of current expenditures that is spent on teacher salaries. The portion of current expenditures that goes toward teacher salaries has been in a slow downward trend, with the teacher salary share falling from 39 percent in 1990–91 to 35 percent in 2003–04. With the projected values, the portion of current expenditures that goes toward teacher salaries continues to fall slowly, to 33 percent in 2015–16. The results of this analysis indicate that the projections of these three time series are consistent.

#### **Projection Accuracy**

Fifteen of the last 16 editions of *Projections of Education Statistics* contained projections of current expenditures and teacher salaries. The actual values of current expenditures and teacher salaries can be compared with the projected values in the previous editions to examine the accuracy of the models.

The projections from the various editions of *Projections* of *Education Statistics* were placed in 1982–84 dollars using the Consumer Price Indices that appeared in each edition.

In most of the earlier editions of *Projections of Education Statistics*, average daily attendance rather than fall enrollment was used as the measure of enrollment in the calculation of the current expenditure per pupil projection. However, projections of current expenditures per fall enrollment were presented in most of these earlier editions, and projections of fall enrollment were presented in all of these earlier editions. As a result, the projected values of both current expenditures per pupil in fall enrollment and current expenditures per pupil in average daily attendance can be compared to their respective actual values.

Similar sets of independent variables have been used in the production of the current expenditure projections presented in the last 13 editions of Projections of Education Statistics, including this one. The one major change is that in all the earlier editions the set of variables included the ratio of the number of students to the population. There have also been some differences in the construction of the variables. First, as noted, average daily attendance was used in most of the previous editions rather than fall enrollment. Second, in Projections of Education Statistics to 1997-98, calendar year data were used for disposable income, the population, and the Consumer Price Index. With the later editions, school year data were used. Third, there have been two revisions in the disposable income time series, the first affecting the Projections of Education Statistics to 2004 and the second, Projections of Education Statistics to 2007. Fourth, in the more recent editions, including this one, the U.S. Bureau of the Census's July 1 number for the population has been used. In the earlier editions, an average of the quarterly values was used. Fifth, in the more recent editions, the U.S. Census Bureau's population projections have been used. In the earlier editions, the population projections came from an economic consulting firm.

There has also been a change in the estimation procedure. In the more recent editions, the AR1 model for correcting for autocorrelation was used to estimate the model. In the earlier editions, ordinary least squares without correcting for autocorrelation was used to estimate the model.

Several commonly used statistics can be used to evaluate projections. The values for one of these, the mean absolute percentage error (MAPE), are presented in table

A-2. MAPEs of expenditure projections are presented for total current expenditures, current expenditures per pupil in fall enrollment, current expenditures per pupil in average daily attendance, and teacher salaries.

To calculate the MAPEs presented in table A-2, the projections of each variable were first grouped by lead time; that is, all the projections of each variable that were a given number of years from the last year in the sample period were grouped together. Next, the percent differences between each projection and its actual value were calculated. Finally, for each variable, the mean of the absolute values of the percent differences were calculated, with a separate average for each lead time. These means are the MAPEs. Table A-2 contains a series of MAPEs for each dependent variable, with a different MAPE for each lead time.

For some editions of the *Projections of Education Statistics*, the first projection to be listed did not have a lead time of 1 year. For example, in *Projections of Education Statistics to 2002*, the first projection to appear was for 1990–91. This projection was calculated using a sample period ending in 1988–89, so it had a lead time of 2 years. The value that appeared for 1989–1990 was from NCES *Early Estimates*. Only those projections that appeared in an edition of *Projections of Education Statistics* were used in this evaluation.

Projections for teacher salaries also appeared in 15 of the last 16 editions of *Projections of Education Statistics*. In these earlier editions, average daily attendance rather than fall enrollment was used as the measure of enrollment. Beginning with *Projections of Education Statistics to 2006*, all the variables for the teacher salary model were placed in log form. With this change in functional form, there was also a change in the way the change in enrollment was measured.

#### Sources of Past and Projected Data

Data from several different sources were used to produce the projections in this report. In some instances, the time series used were made by either combining numbers from various sources or manipulating the available numbers. The sources and the methods of manipulation are described here.

The time series used for current expenditures was compiled from several different sources. For the school years ending in even numbers from 1969–70 to 1975–76, the numbers for current expenditures were taken from

various issues of *Statistics of State School Systems*, published by NCES. For the school years ending in odd numbers during the 1970s, up to and including 1976–77, the numbers were taken from various issues of *Revenues and Expenditures for Public Elementary and Secondary Education*, published by NCES. For the school years from 1977–78 until 2002–03, the data were from the NCES Common Core of Data survey and unpublished data.

For 1974–75 and 1976–77, expenditures for summer schools were subtracted from the published figures for current expenditures. The value for 1972–73 was the sum of current expenditures at the local level, expenditures for administration by state boards of education and state departments of education, and expenditures for administration by intermediate administrative units.

Note that although the data from the different sources are similar, they are not entirely consistent. Also, the NCES data beginning with 1980–81 are not entirely consistent with the earlier NCES numbers, due to differing treatments of items such as expenditures for administration by state governments and expenditures for community services.

An alternative source for current expenditures would have been the U.S. Census Bureau's F-33, which offers statistics at the district level. This level of detail was not needed, however.

For most years, the sources for the past values of average daily attendance were identical to the sources for current expenditures.

Projections for average daily attendance for the period from 2003–04 to 2015–16 were made by multiplying the projections for enrollment by the average value of the ratios of average daily attendance to the enrollment from 1990–91 to 2002–03; this average value was approximately .93.

The values for fall enrollment from 1979–80 to 2002–03 were taken from the NCES Common Core of Data survey. The projections for fall enrollment are those presented in chapter 1 of this publication.

For 1969–70 to 2002–03, the sources for revenue from state sources were the two NCES publications *Statistics of State School Systems* and *Revenues and Expenditures for Public Elementary and Secondary Education*, and the NCES Common Core of Data survey. The methods for producing the alternative projections for revenue from state sources are outlined above.

The estimates for average teacher salaries were taken from various issues of the National Education Association's *Estimates of School Statistics*. These numbers come from their annual survey of states.

The projected values for disposable income, personal taxes and non-tax receipts to state and local governments, and indirect business taxes and tax accruals to state and local governments were developed using projections developed by Global Insight's U.S. Quarterly Model. Projected values of the Consumer Price Index for all urban consumers, which was used for adjusting current expenditures, teacher salaries, revenue from state sources, and the state revenue variables, were also developed using the U.S. Quarterly Model.

The U.S. Census Bureau supplied both the historical and projected values for the population.

The values of all the variables from Global Insight were placed in school-year terms. The school-year numbers were calculated by taking the average of the last two quarters of one year and the first two quarters of the next year.

The Elementary and Secondary School Price Index was considered as a replacement for the Consumer Price Index for placing current expenditures and teacher salaries in constant dollars. This index could not be used because the required projections of the index were not available. There are other price indexes, such as the implicit price deflator for state and local government purchases, which could have been used instead of the Consumer Price Index. These alternatives would have produced somewhat different projections.

Table A-26. Equations for current expenditures per pupil in fall enrollment, estimated average annual salaries of teachers, and education revenue from state sources

Dependent variable					Equation	$\mathbb{R}^2$	Durbin- Watson statistic <sup>1</sup>	Error distribution pattern <sup>2</sup>	Rho	Time period
Current expenditures per pupil	ln(CUREXP)	= - 0.9 +	0.8ln(PCI) (5.7)	+	0.2ln(SGRANT) (2.4)	0.99	1.4	AR(1)	0.80 (10.1)	1969–70 to 2002-03
Estimated average annual salaries	ln(SALRY)	= 7.3 +	0.8ln(CUREXP) (5.7) 1.3ln(ENR/ENR1) (2.4)	+	0.5ln(ENRPOP) (2.1)	0.96	1.58	AR(1)	0.98 (33.0)	1970–71 to 2002-03
Education revenue from state sources per capita	ln(SGRNT)	=3 +	1.3ln(PCI) (17.3)	+	0.7ln(ENRPOP) (5.2)	0.98	1.87	AR(1)	0.49 (3.1)	1971–72 to 2002-03

<sup>&</sup>lt;sup>1</sup>For an explanation of the Durbin-Watson statistic, see J. Johnston and J. Dinardo, Econometric Methods, New York: McGraw-Hill, 1996.

#### Where

CUREXP = Current expenditures of public elementary and secondary schools per pupil in fall enrollment in constant 1982–84 dollars

SALRY = Average annual salary of teachers in public elementary and secondary schools in constant 1982–84 dollars

SGRANT = Local governments' education revenue from state sources, per capita, in constant 1982-84 dollars

PCI = Disposable income per capita in constant 2000 chained dollars

ENRPOP = Ratio of fall enrollment to the population

ENR = Fall enrollment

ENR1 = Fall enrollment lagged one period

NOTE: R<sup>2</sup> indicates the coefficient of determination. Rho measures the correlation between errors in time period t and time period t minus 1. Numbers in parentheses are t-statistics. SOURCE: U.S. Department of Education, National Center for Education Statistics, Elementary and Secondary School Current Expenditures Model, 1969–70 through 2002–03; Elementary and Secondary Teacher Salary Model, 1970–71 through 2002–03; and Revenue Receipts from State Sources Model, 1971–72 through 2002–03. (This table was prepared November 2005.)

<sup>&</sup>lt;sup>2</sup>AR(1) indicates that the models was estimated using least squares with the AR(1) process for correcting for first-order autocorrelation. For a general discussion of the problem of autocorrelation, and the method used to forecast when correcting for autocorrelation, see G. Judge, W. Hill, R. Griffiths, H. Lutkepohl, and T. Lee, *The Theory and Practice of Econometrics*, New York: John Wiley and Sons, 1985, pp. 315–318.